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Cost-constrained project scheduling with task durations and costs that may increase over time: demonstrated with the U.S. Army future combat systems

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NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

COST-CONSTRAINED PROJECT SCHEDULING WITH TASK DURATIONS AND COSTS THAT MAY INCREASE OVER TIME: DEMONSTRATED WITH THE U.S. ARMY FUTURE COMBAT SYSTEMS

by

Roger Grose

June 2004

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13. ABSTRACT (maximum 200 words)

We optimize long-term project schedules subject to annual budget constraints, where the duration and cost of each task may increase as the project progresses. Initially, tasks are scheduled without regard to budgets and the project completion time is minimized. Treating the task durations as random variables, we then use simulation to describe the distribution of the project completion time. Next, we minimize the completion time under budget constraints with fixed task durations, where budget violations are tolerated albeit with penalties. Annual reviews are then introduced, which allow underway tasks to be delayed or monthly budgets to be increased. We obtain estimates of the completion time of the project and its final cost under each of these scenarios. The U.S. Army Future Combat Systems (FCS) is used for illustration. FCS is a suite of information technologies, sensors, and command systems with an estimated acquisition cost of over $90 billion. The U.S. General Accounting Office found that FCS is at risk of substantial cost overrun and delay. We analyze three schedule plans for FCS to identify which can be expected to deliver the earliest completion time and the least cost.

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Submitted in partial fulfillment of the
requirements for the degree of

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We optimize long-term project schedules subject to annual budget constraints, where the duration and cost of each task may increase as the project progresses. Initially, tasks are scheduled without regard to budgets and the project completion time is minimized. Treating the task durations as random variables, we then use simulation to describe the distribution of the project completion time. Next, we minimize the completion time under budget constraints with fixed task durations, where budget violations are tolerated albeit with penalties. Annual reviews are then introduced, which allow underway tasks to be delayed or monthly budgets to be increased. We obtain estimates of the completion time of the project and its final cost under each of these scenarios. The U.S. Army Future Combat Systems (FCS) is used for illustration. FCS is a suite of information technologies, sensors, and command systems with an estimated acquisition cost of over $90 billion. The U.S. General Accounting Office found that FCS is at risk of substantial cost overrun and delay. We analyze three schedule plans for FCS to identify which can be expected to deliver the earliest completion time and the least cost.

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EXECUTIVE SUMMARY

Scheduling an acquisition project subject to time and budget constraints is a challenging management problem. A task not completed on time threatens cost overrun and cascading delay of succeeding tasks, perhaps ultimately delaying the entire project and its fielded capability, and maybe even threatening outright failure of the project.

A project scheduling problem is characterized and viewed as a directed network with an arc depicting each pairwise partial order between completion of a predecessor task and start of a successor task. Planned project completion is governed by the longest directed path in terms of total task durations in this network.

In reality, a task may fail to finish within its planned duration for reasons that cannot be known in advance. One strategy for scheduling the tasks may be more robust to effects of delays than others, even though all schedules are subject to the same constraints. Our objective is to identify the scheduling strategy or strategies that offer the least schedule risk.

This thesis shows how to assess the risks of defense acquisition scheduling under budget constraints. The approach considered is applicable to a wide range of programs that encompass multiple developmental tasks with durations that are subject to uncertainty.

We use the U.S. Army's planned acquisition of the Future Combat Systems (FCS) to demonstrate schedule re-optimization responding to random delays. FCS is a "system of systems" that requires successful completion of many developmental tasks over approximately ten years, where many tasks are dependent on prior completion of other tasks, and many tasks depend on nascent technologies. FCS is ideal to illustrate new concepts of project scheduling under uncertainty.

The U.S. General Accounting Office (GAO) reports that FCS suffers significant risks of cost and schedule growth. These risks might lead to major consequences for the entire U.S. Defense budget. Costs for FCS acquisition include $92 billion (2004 U.S. dollars) to acquire only 14 of the 18 systems that are needed for FCS to achieve initial

operational capability by the year 2010 and $16 billion for the system development and demonstration (SDD) phase alone. In fiscal year 2005, FCS is expected to consume more than 50 percent of the U.S. Army budget for all programs in SDD phases, and over 30 percent of the total budget for research and development and test and evaluation tasks

This thesis examines three plans for scheduling FCS tasks in the SDD phase. The baseline plan is the current schedule. Alternate plan 1 and alternate plan 2 are schedules that were developed based on 2003 GAO recommendations to mitigate schedule risks. Nominal data on FCS tasks provided by the Cost Analysis Improvement Group (CAIG) of the Office of the Secretary of Defense (OSD) are used to optimize the three schedules both with and without annual budget constraints.

Each plan is analyzed under four different scheduling scenarios. Under the first scenario tasks are scheduled without regard to costs and treating the task durations as fixed. Under the second scenario annual budget constraints are not imposed but the task durations in months are generated as random variables using probability distributions in a computer simulation of the entire project. By simulating a schedule many times, we induce project completion time and final project cost as random variables. Summary statistics, such as the mean completion time or mean cost, can be used to evaluate a proposed schedule or to compare multiple schedules.

The third scenario introduces budget constraints by fiscal year, and deterministically optimizes the project schedule by determining for each task the best start month, and selecting from a feasible range of task durations the best planned duration in months, given that monthly task costs may differ depending on start month and duration. In addition, there is a complete project budget by fiscal year for any feasible fiscal year when the project might be completed, and the optimization selects which of these overall project budgets to use while scheduling all the task starts and durations. Although the optimization can insert slack in the project between tasks to satisfy budget constraints, it cannot interrupt a task once that task is started for some given duration. Each fiscal year budget alternative is given as an interval, or "budget band," within which we stipulate that the project is in planned cost control. Because there may be no feasible pattern of task starts and durations that result in spending within

these budget bands, the budgets are expressed cumulatively over the planning horizon, and under- and over-expenditures are tolerated, albeit at a high penalty cost per dollar of violation. The idea is to permit some flexibility by carrying forward any cumulative under- or over-expenditure until it is repaid in some later fiscal year, and encouraging prompt repayment by continuing to penalize any outstanding cumulative violation.

In scenario 4, the budget-constrained, deterministic project schedule optimization is subjected to annual reviews. In each of these successive fiscal year reviews, any task already underway and not yet finished may suffer cost growth and/or duration delay. Such events are influenced by the degree of risk assigned to each task, and we decide whether and by how much to inflate cost and duration of tasks under review by a random simulation.

The results for all four scenarios are summarized in the table below. For example, using the baseline plan under scenario 1 as our reference schedule, alternate plan 1 under scenario 2 leads to an estimated project delay of seven percent. When budget constraints with annual reviews are imposed, this delay grows to approximately 37 percent. For FCS, a 37 percent delay corresponds to approximately four years, where a one-year delay has been estimated by the GAO to add between $4 billion and $5 billion to the total acquisition cost.

In the absence of budget constraints, developing critical technologies to a production-suitable readiness level prior to other tasks (alternate plan 1) leads to project completion faster than the baseline plan. When budget constraints are added, this plan maintains its advantage although it is subject to delays similar to the baseline plan.

Development of the C4ISR system up front (alternate plan 2) presents a very different schedule risk. Although this plan stands out as the least desirable option in the absence of budget constraints, it emerges as the most favorable option when these constraints are imposed. Because budget constraints are a reality in defense acquisition, alternate plan 2 evidently presents the least schedule risk.

	Program Delay Relative to Current Army Plan by Scheduling Scenario (%)			
	Without Budget Constraints		With Budget Constraints	
Schedule Plan	**Scenario 1** Without Schedule Simulation	**Scenario 2** With Schedule simulation	**Scenario 3** Without Annual Reviews	**Scenario 4** With Annual Reviews
Baseline plan: Proceed with current Army planned schedule.	0	27	10	39
Alternate plan 1: Mitigate high risk technologies prior to other tasks.	-2	7	8	37
Alternate plan 2: Develop the C4ISR system prior to other systems.	9	18	21	23

Overview of Percentage Schedule Delay by Plan and by Scenario
Each numeric entry indicates the percentage delay that a program experiences compared to what the FCS program office has forecast in the baseline plan and scenario 1. E.g., the baseline plan under scenario 4 suffers a 39 percent delay.

Although we use hypothetical data, important features of the schedule plans such as their relative schedule risk emerge. This thesis provides a general framework in which acquisition schedules can be analyzed and compared. The framework provides simple metrics for comparisons of multiple plans with uncertain task durations and budget constraints. Schedules produced using this framework show how program managers can optimally respond to re-schedule their programs when task durations and costs are not only uncertain at the start of the program, but subject to unpredictable annual changes.

LIST OF ACRONYMS

AFSS	Advanced Fire Support System
ARV	Armed Reconnaissance Vehicle
BLOS	Beyond Line of Sight
C2	Command and Control
C4ISR	Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance
CAIG	Cost Analysis Improvement Group
CDF	Cumulative Distribution Function
CLU	Container Launch Unit
CPLEX	Optimization Solver developed by ILOG
CPM	Critical Path Method
CV	Command Vehicle
DoD	Department of Defense
DRR	Design Readiness Review
FCS	Future Combat Systems
FY	Fiscal Year
GAMS	General Algebraic Modeling System
GAO	General Accounting Office
GUB	Generalized Upper Bounds
ILP	Integer Linear Programming
INFORMS	Institute for Operations Research and the Management Sciences
IOC	Initial Operational Capability
IOT&E	initial operational test and evaluation
LOS	Line of Sight
LAM	Loitering Attack Missile
LWIR	Long-Wave Infrared
MDAP	Major Defense Acquisition Program
MULE	Multi-functional Utility Logistics and Equipment vehicle
MWIR	Medium Wave Infrared
NLOS	Non-Line of Sight
NP	Non-Polynomial
OSD	Office of the Secretary of Defense
OSL	Optimization Subroutine Library
PAM	Precision Attack Missile
PDF	Probability Distribution Function
PEO	Program Executive Office
PERT	Program Evaluation and Review Technique
RCPSP	Resource Constrained Project Scheduling Problem
RSTA	Reconnaissance, Surveillance and Target Acquisition
S&T	Science and Technology
SDD	System Design and Development

SUAV	Small unmanned air vehicle
T&E	Test and Evaluation
TRL	Technology Readiness Level
TV	Television
UAV	Unmanned Aerial Vehicle
UGV	Unmanned Ground Vehicle

I. INTRODUCTION

This thesis presents an approach for analyzing defense acquisition scheduling plans with uncertain task durations, and subject to annual budget constraints on monthly task costs. The paradigm that we consider is applicable to a wide range of programs which require multiple developmental tasks with timelines that are subject to uncertainty. Tasks not completed within their designated timelines pose risks to an acquisition program. These risks consist of cost overruns, cascading delays as some tasks cannot begin before others finish, the lack of fielded capability on a timely basis, and even the outright failure of the program itself.

In acquisition planning, the tasks that comprise a project are scheduled as a network, recognizing their interdependencies. A task may fail to finish within its allotted time for reasons that cannot be known in advance. This uncertainty arises from resource unavailability, the need to modify start times due to previous tasks not having finished on schedule, changes in project scope, and other factors (Herroelen and Leus, 2004).

This thesis considers two types of scheduling problems. The first is where budgets and activity costs are not considered, but task durations are described using probability distribution functions in a computer simulation of the entire project. From this analysis, it is possible to characterize the completion time and final cost of the project as random variables. Summary statistics, such as the mean completion time or mean cost, can be used to evaluate a proposed schedule or to compare multiple schedules.

The second scheduling problem considered is the resource-constrained project scheduling problem (RCPSP) that has been considered since the earliest days of operations research (Kelly, 1963). In this thesis, the optimal start times and durations of future tasks are determined and scheduled subject to both annual and overall budget constraints.

The U.S. Army's planned acquisition of the Future Combat Systems (FCS) is used to demonstrate the schedule-analysis approach developed in the thesis. FCS is a networked "system of systems" that requires the successful completion of many developmental tasks in order to bring it to fruition. An extended acquisition period

(approximately ten years), reliance on nascent technologies, and interdependence among developmental tasks make FCS well suited for illustration of the concepts of project scheduling under uncertainty.

A. ACQUISITION OF THE U.S. ARMY FUTURE COMBAT SYSTEMS

FCS is a networked system of systems that is under development to enable the U.S. Army and the Department of Defense (DoD) project overwhelming military power anywhere in the world. An overview of the systems that comprise FCS is shown in Figure 1. FCS includes a family of air-deployable manned ground vehicles, two unmanned aerial vehicles and three unmanned ground vehicles. An unattended ground sensor and a non-line of sight rocket launch system complement these systems. All of these systems are integrated within a sophisticated command, control, communications, computers, intelligence, surveillance, and reconnaissance (C4ISR) architecture. Detailed descriptions of the systems that comprise FCS are provided in Appendix A.

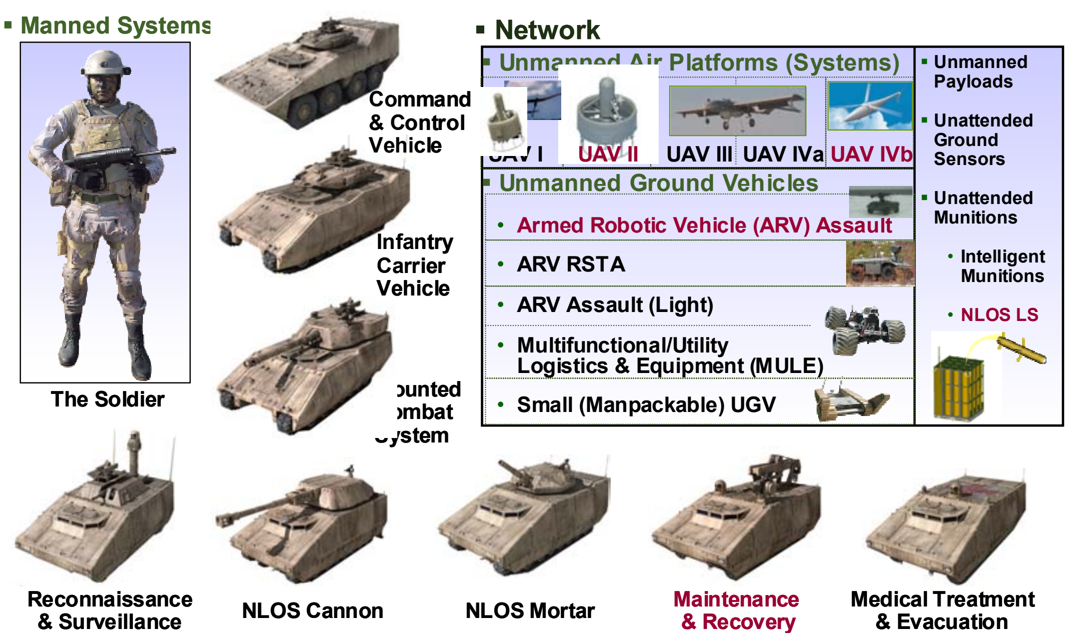

Figure 1. Future Combat Systems Composition (after Brady, 2003). The scope and magnitude of the FCS program is unprecedented in U.S. Army acquisition.

U.S. defense acquisition programs, including FCS, operate within the Defense Acquisition Framework, which is illustrated in Figure 2. Milestone B represents the formal starting point of the acquisition process, which begins with the System Development and Demonstration (SDD) phase and ends with the fielding of a fully operational system. FCS entered the SDD phase in May of Fiscal Year (FY) 2003, and is scheduled for full operational capability in fiscal year 2013 (U.S. GAO, 2003).

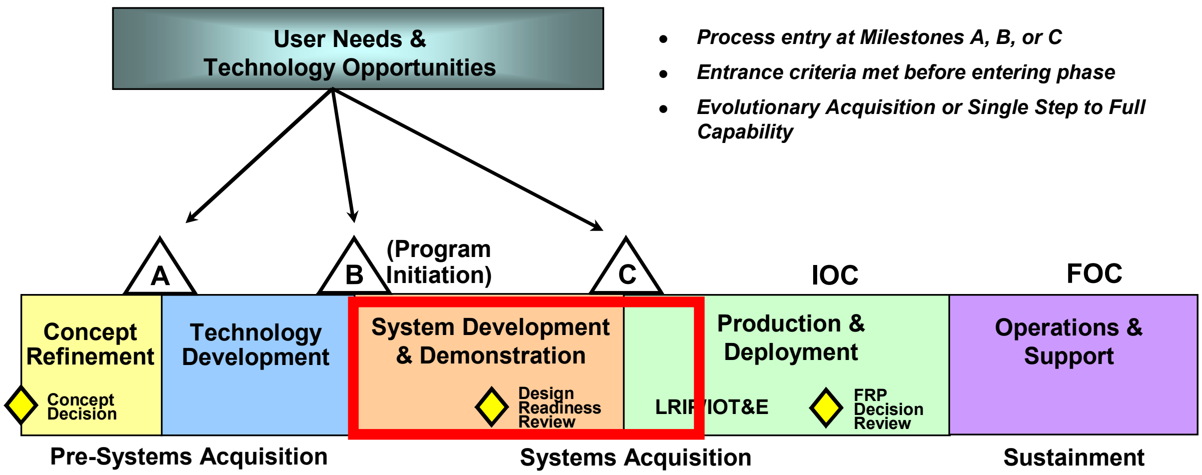

Figure 2. Defense Acquisition Framework (after Wynne, 2003).
The thesis focuses on the area outlined by the bold rectangle.

Normally, defense acquisition programs have only one Milestone B pass-through, but FCS will be admitted through this milestone piecemeal due to the challenges posed by the development of its many systems. At Milestone C, attention will shift toward system integration and demonstration. Normally, system integration uses prototype articles, but in the case of FCS a full-system prototype assembly will not be available before the production decision has been made. Instead, FCS will rely on simulation-based acquisition, and combined developmental and operational testing. An independent initial operational test and evaluation (IOT&E) using an incomplete prototype is scheduled for 2008 (Welch, 2003).

Francis (2004) cited cost estimates for FCS in his testimony before Congress. They include $92 billion (2004 U.S. dollars) to acquire only 14 of the 18 systems that are needed for FCS to achieve initial operational capability by the year 2010 and $16 billion for the SDD phase alone. In fiscal year 2005, it is expected that FCS will consume more

than 50 percent of the U.S. Army budget for all programs in SDD phases, and over 30 percent of the total budget for research and development (R&D) and test and evaluation (T&E) tasks.

The complexity of FCS and an aggressive developmental schedule introduce risks into its acquisition program. At the start of the SDD phase in 2003 about three-quarters of its critical technologies were classified as immature (Francis, 2004). FCS acquisition planning is based extensively on developmental concurrency across the different phases of the project. Francis (2004) outlines the myriad of tasks that must be coordinated in order for this strategy to be successful:

- A specialized C4ISR network must be developed for FCS.
- Fourteen major weapon systems and platforms must be designed and integrated simultaneously with other systems, subject to physical limitations.
- At least 53 technologies that are considered critical to achieving required performance capabilities must be matured and integrated.
- At least 157 Army and joint-forces systems must also be adapted to interoperate with FCS, which will require the development of nearly a hundred new network interfaces.
- An estimated 34 million lines of software code will be required to operate FCS. This is nearly five times the software required for the Joint Strike Fighter, which had the largest software requirement of any Department of Defense acquisition prior to FCS.

It is difficult to ensure that the development of a new technology will be completed within a specified time period. As a system of systems, FCS is especially vulnerable to the cascading effects of schedule overruns from projects to follow-on tasks. Francis (2004) observes that the completion of FCS as planned is unlikely given the many opportunities for delay in development of its constituent systems. He estimates that a one-year delay late in FCS development could add $4 billion to $5 billion to the total cost. Non-budgetary costs, such as the effect on warfighting capability of not having a fielded system on schedule, are more difficult to quantify but are no less important.

B. SCHEDULE OPTIONS AND SCHEDULE RISK

The term *schedule risk* refers to the costs of schedule overruns balanced by their likelihood of occurrence. Planners may be presented with a set of options for scheduling the range of tasks that comprise a large acquisition project. These options must abide by a common set of temporal and fiscal constraints. They should also reflect the inherent uncertainty of the completion time of a developmental task. A rational planner seeks to assess the schedule risks of each option, and to select the option that poses the least risk.

Significant knowledge demonstration often occurs late in development and early in production of major defense acquisition programs (MDAPs), of which FCS is an example. Integration of developed components into a system of systems has the highest schedule risk. Welch (2003) observes that the unusual complexity of FCS exposes it to higher integration schedule risk than normally expected of a MDAP. In particular, FCS is susceptible to "late cycle churn" due to the anticipated need to fix problems discovered late in development. Francis (2004) identifies the following factors that dispose FCS to late cycle churn:

- Technology development that is expected to continue through to the production decision (Milestone C);
- Technology development that will still be ongoing at the design readiness review (DRR) in July 2006, putting at risk the stability of ongoing system integration;
- The planned move into production in December 2007 while technology development and system integration are continuing and the first prototypes are being delivered;
- The planned final production decision in November 2008 that will be made before some technologies will have reached their required maturation and an integrated system demonstration will remain to be done;
- The planned start of production delivery in early 2010 before the Army has completed the first full demonstration of FCS as an integrated system;
- The planned full rate production decision in mid-2013 while testing and demonstration are continuing.

The FCS Program Executive Office (PEO) has prepared a "baseline" project plan for the SDD phase that governs current acquisition policy. Several alternate project plans were developed by the General Accounting Office (U.S. GAO, 2003) to mitigate FCS

schedule risks. The baseline plan and two of the alternate plans proposed by GAO are examined in this thesis. The first alternative is based on addressing risky technologies prior to undertaking further development. The second alternative is focused on the development of the C4ISR infrastructure prior to all other systems. Each of the three plans is discussed separately below.

1. Baseline Plan

The baseline plan develops all major sub-systems concurrently, rather than developing one first to set the development context for follow-on systems. Details of the baseline plan can be found in Appendix B. The FCS PEO has acknowledged that this plan is ambitious, and that the program was not ready for system development and demonstration when it was approved (Francis, 2004).

2. Alternate Plan 1

Alternate plan 1 modifies the baseline plan by first developing critical technologies that are not at a production-suitable readiness level at the start of the SDD phase. Most of the lower-risk FCS technologies have already been developed from early proof of concept experiments to a prototype demonstration in test environments prior to the SDD phase (Francis, 2004). For the purposes of this plan, prototypes must be demonstrated in mission environment before it can be considered ready for integration (Wynne, 2003). Risk-mitigation strategies have already been developed by the FCS PEO for the high-risk technologies that are yet to demonstrate a prototype in a mission environment (Welch, 2003). The duration of these mitigation tasks are listed in the schedule for alternate plan 1, provided in Appendix B. Once risk mitigation activities are complete, alternate plan 1 proceeds as scheduled for the baseline plan. The advantage of this approach is that test and integration tasks occur later in the schedule, with reduced schedule risk compared to the baseline plan.

3. Alternate Plan 2

Alternate plan 2 modifies the baseline plan by prioritizing the development of C4ISR tasks before all others. The C4ISR components are believed to pose the greatest schedule risks in FCS development due to the scope and complexity of their requirements. Software engineering estimates for C4ISR components anticipate the need for approximately 16 million lines of code, of which more than half will be new code (Welch, 2003). This huge undertaking is vulnerable to cost and schedule overruns. By placing early priority on these components, subsequent C4ISR test and integration tasks should entail less risk than in the baseline plan. Full details of alternate plan 2 are in Appendix B.

C. PURPOSE OF THESIS

The purpose of this thesis is to schedule FCS tasks in the SDD phase to minimize the total project duration, which is the elapsed time from the start of the first task to the end of the last task. The last task is assumed to be achievement of the FCS initial operational capability (IOC). Minimization is subject to periodic (annual) and overall budget constraints, and to temporal ordering relationships among the tasks. Because all schedules operate under a common set of constraints, minimization of the total project duration is taken to be synonymous with minimization of schedule risk. Together, the cost constraints, task network, and task duration attributes constitute data that are input to the minimization procedure. Although the thesis is focused on applying this procedure to FCS, by varying the inputs it can be applied to any scheduling problem of similar structure.

Minimization of the project completion time falls within the scope of the Resource Constrained Project Scheduling Problem (RCPSP), which has a long history in operations research. Chapter II provides a brief review of the RCPSP literature for both deterministic and random task durations. Task durations are modeled as random variables, and an algorithm for solving the unconstrained scheduling problem is described in Chapter III. In Chapter IV an integer linear program (ILP) is formulated for the scheduling problem subject to budgetary constraints. Chapter IV also presents the annual review simulation. The results of applying the ILP to nominal data for FCS tasks in the

SDD phase are presented in Chapter V. Conclusions and recommendations for further research are discussed in Chapter VI.

Nominal FCS task information was provided by the Cost Analysis Improvement Group (CAIG) of the Program Analysis and Evaluation (PA&E) branch of the Office of the Secretary of Defense (OSD), who sponsored this thesis.

II. RELATED RESEARCH

A. THE RESOURCE-CONSTRAINED PROJECT SCHEDULING PROBLEM

The Resource Constrained Project Scheduling Problem (RCPSP) is to find task starting times that minimize the total project duration subject to resource and temporal constraints. Tasks are configured in a network that defines precedence relationships. Project duration is the length of the longest path through the network, which is also known as the *critical path*. Minimization of this length is subject to time constraints on tasks, and to resource constraints that can be formulated in a number of ways: on the total cost of the project, on the annual costs of the project, etc. A review of methods for solving formulations of the RCPSP can be found in Demeulemeester and Herroelen (2002).

The use of linear programming to solve scheduling problems has a long history in operations research (Bowman, 1958). ILP approaches followed with the pioneering work of researchers including Senju (1968), and Pritsker, Watters and Wolfe (1969). The RCPSP is known to be non-polynomial (NP)-hard in general, which suggests that polynomial-time algorithms for solving this problem are unlikely to exist (Ullman, 1975). Most algorithms use heuristics to reduce problem size and complexity prior to initiating enumeration of feasible solutions. Integer linear programming (ILP) based on branch-and-bound or other polyhedral-based techniques can then be used to identify the optimal feasible solution.

Probabilistic modeling of task durations began to appear in the scheduling literature in the late 1950s and 1960's. During that time, the Program Evaluation and Review Technique (PERT) and the Critical Path Method (CPM) were developed. Seminal papers include Malcolm, Roseboom, Clark, and Fazar (1959) in which PERT was first developed for the Polaris Fleet Ballistic Missile program, and Kelly (1961) which provides much of the mathematical basis for its later use. CPM and PERT have since combined to form a single method that is among the most widely used operations research techniques in project management.

Drawbacks to PERT are that it does not allow for the inclusion of resource constraints, and that it only considers the critical path formed when all tasks are fixed to their mean duration (Weist, 1964). Significantly, a task not on the PERT critical path using its mean duration may be on the critical path with positive probability when its duration is treated as a random variable. PERT estimates of project duration are generally optimistic. Fulkerson (1962) demonstrates this optimism using tasks that are modeled as discrete random variables. He shows that PERT networks using only mean durations always under-estimate the project finish time relative to treating the durations as random variables.

Dodin (1984) reports upper and lower bounds on the project duration where task durations are modeled as independent random variables. The independence assumption is used to invoke the Central Limit Theorem to justify treating the project duration as approximately normally distributed. While this assumption lends tractability, it is rarely true in practice and it can give misleading results.

B. PROJECT SCHEDULING UNDER UNCERTAINTY

A number of approaches have been developed for solving the RCPSP with stochastic inputs. Seminal papers include Babbar, Tintner, and Heady (1955), Tintner (1955), Tintner (1960), and Sengupta, Tintner, and Morrison (1963). Factors that influence task duration are treated as random variables with distributions that may not be completely known (Herroelen, Reyck, and Demeulemeester, 1998). These factors include resources availability, scheduling of deliveries, modification of due dates, and changes in project scope that may imply the cancellation or addition of future tasks (Herroelen and Leus, 2004).

Although stochastic modeling lends greater realism to the RCPSP, it also increases its analytical complexity. Instead of minimizing the project critical path length, objectives such as minimizing the *expected* project critical path length are often considered, or minimizing expected costs that include penalties for violating constraints (Gutjahr, Stauss, and Wagner 2000).

With many interdependent tasks represented in a project network, and task durations that are interdependent, the probability distribution of the total project duration is difficult to characterize (Yang, Geunes, and O'Brien, 2001). An independence assumption is often made to allow for tractable analysis, but as noted above this assumption is not realistic. Nonetheless, some insight can be gained by adopting an independence assumption. For example, in a deterministic setting an optimal schedule may be found, but it may fail to be robust to small changes in its underlying data. An optimal deterministic schedule typically has insufficient slack to remain optimal (or even feasible) in an uncertain setting, and thus lacks robustness (Herroelen, 2004). Introducing randomness in even a simplified manner can reveal this property.

In planning a large, multifaceted project, managers would want to have the flexibility to change their scheduling decisions as the project evolves. Under *full dynamic scheduling* this can be done during project execution, at decision points consisting of the completion times of tasks (Igelmund and Radermacher, 1983). These decision points are stochastic, because they depend on the task durations. This thesis adopts a simpler form of dynamic scheduling whereby the decision points are the ends of fiscal years, which are deterministic. All tasks that have not completed by the end of a given fiscal year are eligible for rescheduling. This brings the realism of dynamic scheduling into the RCPSP formulation that is described in Chapter IV.

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III. UNCONSTRAINED STOCHASTIC SCHEDULE ANALYSIS

This chapter presents an approach to studying the completion time of a project with random task durations, but without resource constraints. Task durations are modeled as independent random variables having three-parameter Weibull distributions. Properties of the three-parameter Weibull distribution and a model selection procedure that can be used to guide its application are presented in Section A. In a simulation exercise a full set of task durations is generated, and an unconstrained reaching algorithm is used to identify the completion time of the project, which is the longest path in a network from the first to last task. This algorithm is described in Section B. The simulation was coded in Java, which is presented in Appendix C.

A. STOCHASTIC MODELING OF TASK DURATIONS

The three-parameter Weibull distribution is often used to model the duration of developmental tasks for cost estimation and planning. It has the following density function:

$$f\left(x;\, d_{Min}, \alpha, \beta\right) = \begin{cases} \dfrac{\alpha}{\beta}\left(\dfrac{x - d_{Min}}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x-d_{Min}}{\beta}\right)^{\alpha}}, & x \geq d_{Min} \\ 0, & x < d_{Min}\,. \end{cases}$$

The three nonnegative parameters $d_{Min}, \alpha,$ and β uniquely define the distribution. The location parameter, d_{Min}, is a guaranteed lower bound on the random variable X which represents the task duration. Parameters α and β are associated with the shape and scale of the distribution, respectively. Both d_{Min} and β are measured in the same time units as X, but α is unitless.

Although the three-parameter Weibull distribution is defined for any value of α greater than zero, for modeling task durations it is desirable to restrict α to values greater than one. This ensures that the density achieves its maximum at a value $x = x_M$ that is

strictly greater than d_{Min}. The maximum likelihood value x_M is also known as the *mode*, or most likely value, of the distribution.

Through proper selection of its parameters, the three-parameter Weibull distribution can emulate intuitive features of task duration. For example, large deviations from the norm are more common in the positive than in the negative direction, which is reflected in the positive skewness of the three-parameter Weibull distribution. And, many developmental tasks cannot finish in less than a minimum time, which is represented by d_{Min}.

Selection of a three-parameter Weibull distribution to model task duration is equivalent to specifying its parameters. This specification is largely subjective, depending on the judgment of the planner for the task in question. Miller (2003) describes a convenient procedure for specifying the parameters of a three-parameter Weibull distribution from intuitive features of the task duration. The analyst needs to provide the following information:

- A value for the duration mode, x_M;
- Categorization of the risk level as High, Medium, or Low.

Each risk level is assigned to fixed values of two attributes of the task duration, which together with x_M are sufficient to determine all three parameters of the model. Attribute $R_M = x_M / d_{Min}$ is the ratio of the mode to the guaranteed duration, and $P_M = P(X > x_M)$ is the probability that the duration exceeds the mode. Table 1 shows the association of risk levels to values of these two attributes. The values shown in Table 1 were chosen by Miller (2003) to reflect historical evidence of task durations in cost estimation. He suggest using the following guidelines for selection of the schedule risk level:

- **High risk** for unprecedented tasks;
- **Medium risk** for development and some integration tasks;
- **Low risk** for routine tasks that are well understood.

Risk Level	R_M	P_M
High	1.25	0.8
Medium	1.20	0.7
Low	1.15	0.6

Table 1. Association of Risk Levels to Attributes of the Three-Parameter Weibull Distribution (after Miller, 2003).

The three-parameter Weibull distribution can be defined using either the triplet (α, β, d_{Min}) or the triplet (x_M, R_M, P_M). Table 2 shows the mapping between these equivalent descriptions of the three-parameter Weibull distribution.

Attributes	Parameters
$x_M = d_{Min} + \beta\left(1 - \frac{1}{\alpha}\right)^{1/\alpha}$	$\alpha = \frac{1}{1+\ln(P_M)}$
$R_M = \frac{x_M}{d_{Min}}$	$\beta = \frac{x_M(1-1/R_M)}{[-\ln(P_M)]^{1+\ln(P_M)}}$
$P_M = e^{-(1-1/\alpha)}$	$d_{Min} = \frac{x_M}{R_M}$

Table 2. Association Between Attributes and Parameters of the Three-Parameter Weibull Distribution

Example. A task is identified as having medium risk, and its most likely duration is $x_M = 36$ months. From Table 1, $R_M = 1.20$ and $P_M = 0.7$. The model parameters are found using the mapping in Table 2:

$$\alpha = \frac{1}{1+\ln(0.7)} = 1.554, \quad \beta = \frac{36(1-1/1.20)}{[-\ln(0.7)]^{1+\ln(0.7)}} = 11.65, \quad d_{Min} = \frac{36}{1.20} = 30.0\,.$$

Simulation of three-parameter Weibull variates is easily done by applying the inverse cumulative distribution function (CDF) to uniform variates. If U has a uniform distribution on the interval $[0,1]$, then X has a three-parameter Weibull distribution with parameters (α, β, d_{Min}), where

$$X = d_{Min} + \beta[-\ln(U)]^{1/\alpha}\,.$$

A drawback to the use of the three-parameter Weibull distribution to model task durations is that its upper bound is infinite. Not only does this fail to reflect practical realities of project development, it also creates problems for simulation of task durations under budget constraints, which is expored in Chapter IV. To ensure that a task finishes within a fixed allotment of time, the thesis research uses three-parameter Weibull distributions that are truncated at the 90th percentile. The truncation point, d_{Max}, is the maximum allowable duration. It is calculated as follows:

$$d_{Max} = d_{Min} + \beta[-\ln(0.1)]^{1/\alpha}\,.$$

Variates from the truncated distribution are generated as

$$X = d_{Min} + \beta[-\ln(1-.9U)]^{1/\alpha}\,.$$

B. THE UNCONSTRAINED REACHING ALGORITHM

The unconstrained reaching algorithm searches over the schedule to identify the completion time of the project, given a task network with fixed durations. The completion time is characterized by the length of the longest path from the start to finish nodes in the network. The unconstrained reaching algorithm, shown in Figure 3, is one

of the simplest network algorithms, as the following pseudo-code for it demonstrates, and it can be executed quickly on a computer (Ahuja, Magnanti, and Orlin, 1993):

```
// initialize
distance[start node] = 0;
loop over all nodes i {
   loop over all arcs j from node i {
      distance[i] = max (cost[i,j]+1)
      pred[j] = 0
   }
}

// search for longest path
loop over all nodes i {
   loop over all arcs j from node i {
      if distance[j] > distance [i] + cost[i,j] then {
         distance[j] = distance[i] + cost[i,j];
         pred[j] = i;
      }
   }
}

// output
longest path length = distance[last node]
```

Figure 3. Unconstrained Reaching Algorithm Pseudo-Code

Using the $O-$notation introduced by Bachmann (1894), the unconstrained longest path is solved by the reaching algorithm on an m-task project network in $O(m)$ steps for an acyclic digraph. In other words, the number of steps grows at an approximately linear rate as m increases. This complexity cannot be improved because any algorithm must examine every task, which itself takes $O(m)$ steps (Weisstein, 2004).

A simulation experiment was conducted to compare the three FCS project plans (baseline plan, alternate plan 1, and alternate plan 2) using unconstrained schedule analysis. For each instance, the simulation samples new task durations from their probability distributions and the finish time of the last task is recorded. The simulation is repeated for 60,000 iterations. As individual project tasks have random durations, the project finish time is itself a random variable that can be observed as the simulation progresses. Results of the simulation are presented in Chapter V.

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IV. BUDGET-CONSTRAINED DETERMINISTIC OPTIMIZATION MODEL

Unconstrained schedule analysis, while lending insight, does not deliver a project schedule, and its lack of budget constraints is not realistic. The incorporation of budget constraints presents the need for an optimization model to identify the shortest project completion time. The approach adopted in this thesis is an integer linear program (ILP) that partially enumerates feasible task schedules, selecting those that minimize the length of the project critical path while observing annual and project budget constraints. Unlike the unconstrained analysis, the task durations are deterministic, chosen from a range of admissible durations.

A. MODEL STATEMENT

An expository description of the budget-constrained schedule optimization model is presented below. For the sake of clarity not all combinations of indices in mathematical expression that follow are necessarily valid. The complete scheduling formulation is given in Appendix D.

1. Index Use

$y \in Y$	Fiscal years that can be covered by the project. There are 17 years considered from 2003, 2004, …, 2020.
$yh \in Y$	Historical fiscal year.
$yf \in Y$	Project finish fiscal years.
$i \in I$	All task within a project plan.
$j \in I$	All task within a project plan that follow task i.
$\ell \in I$	Last task in schedule that marks the completion of the project.
$m \in M$	Possible month within the planning horizon.
$m \in M(y)$	month in fiscal year y

$s_i \in S_i \subseteq M$ Start month for task *i*

$d_i \in D_i$ Task *i* duration in months.

$1 \leq p_i \leq d_i$ Period of ongoing task *i*.

2. Data [units]

$\underline{budget}_{y,yf}$ Lower cost range during fiscal year *y* if program finished in fiscal year *yf* [cost]

$\overline{budget}_{y,yf}$ Upper cost ranges during fiscal year *y* if program finished in fiscal year *yf* [cost]

$cost_{id_ip_i}$ Cost of ongoing task *i* with duration *d* during elapsed month *p* [cost]

pen_under Cost per unit of negative cumulative budget range violation [months/cost]

pen_over Cost per unit of positive cumulative budget range violation [months/cost]

3. Variables [units]

$X_{is_id_i}$ Binary variable, which is set to 1 if task *i* is started in month *s* with duration *d* and set to 0 otherwise [binary].

Q_{yf} Binary variable, which is set to 1 if finish year of program is year *yf*, and set to 0 otherwise [binary].

$UNDER_y$ When expenditures through fiscal year *y* are compared with desired lower ranges on total budgets, this variable measures lower-range violations [cost].

$SLACK_y$ When expenditures through fiscal year *y* is compared with desired lower and upper ranges on total budgets, this variable measures unspent funds below upper-range violation [cost].

$OVER_y$ When expenditures through fiscal year y are compared with desired upper ranges on total budgets, this variable measures upper-range violations [cost].

4. Formulation

$$\underset{\substack{X,Q\\UNDER\\SLACK\\OVER}}{MIN} \quad \sum_{s_\ell, d_\ell} (s_\ell + d_\ell - 1) X_{\ell s_\ell d_\ell} + \sum_y \left(pen\_under\, UNDER_y + pen\_over\, OVER_y \right) \qquad (F1)$$

$$s.t. \qquad \sum_{s_i, d_i} X_{i s_i d_i} = 1 \qquad \forall i \in I \qquad (F2)$$

$$X_{\ell s_\ell d_\ell} \le Q_{yf} \qquad \forall yf, s_\ell, d_\ell \qquad (F3)$$

$$\sum_{yf} Q_{yf} = 1 \qquad (F4)$$

$$\sum_{yf, m, i, s_i, d_i} cost_{id_i(m - s_i + 1)} X_{i s_i d_i} + UNDER_y + SLACK_y - OVER_y = \sum_{yh, yf} \left(\overline{budget}_{yh,yf} \right) Q_{yf} \qquad \forall y \qquad (F5)$$

$$SLACK_y \le \sum_{yh, yf} \left(\overline{budget}_{yh,yf} - \underline{budget}_{yh,yf} \right) Q_{yf} \qquad \forall y \in Y \qquad (F6)$$

$$\sum_{s_i, d_i} X_{i s_i d_i} \ge X_{j s_j d_j} \qquad \forall (i, j), \forall s_j, d_j \; (F7)$$

$$X_{i s_i d_i} \in \{0,1\} \qquad \forall i, s_i, d_i \qquad (F8)$$

$$Q_{yf} \in \{0,1\} \qquad \forall yf \qquad (F9)$$

$$UNDER_y \ge 0;\; SLACK_y \ge 0;\; OVER_y \ge 0 \qquad \forall y \qquad (F10)$$

5. Description of the Model

The objective function (F1) expresses total planned project duration in months, plus an elastic violation term for any violation of budget ranges over the planning horizon.

Constraints:

(F2) We must select exactly one start month and duration in months for each task.

(F3) Each constraint permits the last project task to be completed in a fiscal year only if that fiscal year has been selected for project completion.

(F4) We must select exactly one year for project completion.

(F5) Sum the costs of all tasks active before or during year *y*, and determine any difference between this actual planned expenditure and the intended cumulative maximum budgets over the same epoch.

(F6) For each year, limit the slack to be less than the difference between the maximum and minimum program budget for that year.

(F7) Examine every task and ensure that it does not start until all of its predecessors have finished.

(F8) Selections of $X_{is_id_i}$ are to be binary.

(F9) Selections of Q_{yf} are to be binary.

(F10) Slack, under-expenditure and over-expenditure must be non-negative.

6. Computer Implementation

The optimization model has been implemented in General Algebraic Modeling language GAMS (Brook, Kendrick, Meeraus, Raman, 1998), the code for which is provided in Appendix E.

B. TASK DURATIONS AND COSTS

In the optimization model, the deterministic range of durations in months for each task is modeled by a truncated three-parameter Weibull distribution, as described in Chapter III. Each task of non-zero duration in the schedule incurs a cost in terms of time and materials. Zero-duration tasks are used to represent project milestones, which prevent further progress until all preceding tasks have been completed. The point estimate for task cost is then spread across each month in the task duration according to a method developed by the sponsor using the Rayleigh distribution (Jarvis, 2001), as indicated below:

$$PeriodCostSpread_p = \frac{Budget}{0.97}\left[\exp\left(\frac{(p-1)^2 \ln(0.03)}{d^2}\right) - \exp\left(\frac{p^2 \ln(0.03)}{d^2}\right)\right].$$

The derivation of this formula from the Rayleigh cumulative distribution function (CDF) is as follows. First, note that the Rayleigh CDF is expressed as

$$F(p) = 1 - \exp\left(\frac{-p^2 C}{d^2}\right).$$

Next, define

$$\lambda_j = F(p_j) - F(p_{j-1}) = \exp\left(\frac{-p_{j-1}^2 C}{d^2}\right) - \exp\left(\frac{-p_j^2 C}{d^2}\right), \quad j = 1, \ldots, d.$$

In the case where $p_0 = 0, p_1 = 1, \ldots, p_d = d$ the above formula is more simply expressed:

$$\begin{aligned}
\lambda_1 &= 1 - \exp\left(\frac{-C}{d^2}\right) \\
\lambda_2 &= \exp\left(\frac{-C}{d^2}\right) - \exp\left(\frac{-4C}{d^2}\right) \\
&\vdots \\
\lambda_d &= \exp\left(\frac{-(d-1)^2 C}{d^2}\right) - \exp(-C),
\end{aligned}$$

where $\lambda_1 + \lambda_2 + \ldots + \lambda_d = 1 - \exp(-C)$. By assumption, $1 - \exp(-C) = 0.97$, from which $C = -\ln(0.03) = 3.5066$.

This method of spreading task costs over the months of its duration using a Rayleigh distribution is widely used in budget phasing by the research sponsor, and is an attempt to model historical experience with program expenditure (Jarvis, 2001). This is in contrast to most scheduling approaches that assume task budgets are expended at a uniform rate throughout the task duration.

C. PROJECT BUDGET

Upper and lower limits of the overall planned project budget must be estimated for each fiscal year of the project duration. Separate estimates must be prepared for every feasible project finish year. The same method, based on the Rayleigh distribution, is used for spreading task costs over the desired project duration.

Over-expenditure or under-expenditure is permitted in the model, but is penalized. To minimize penalties, it is desirable that in following periods corrections are made to slow progress and recover from over-spending, or allow more tasks to occur in the case of under-spending.

D. SIMULATION OF ANNUAL REVIEWS

The budget-constrained, deterministic project schedule optimization is subjected to annual reviews. A simulation is used at the start of each fiscal year of the project to resample parameters for tasks underway and not yet completed. The idea is to capture the unpredictable nature of tasks' completion times. Tasks planned further into the future are more likely to require changes to their schedules. Moreover, if future tasks plan to use advanced technologies (some of which are still in development) then such changes are almost certain (Francis, 2004).

The initial solution, at time zero, is equivalent to the optimal schedule with no re-sampling. At each subsequent time step the program optimally schedules underway tasks given that decisions for completed tasks have already been made. As each fiscal year boundary is reached a "program review" is conducted. Tasks already started and still in progress are re-sampled to determine if the remaining task durations and costs will be greater than that planned, or remain unchanged. The flowchart in Figure 4 shows the annual review re-sampling mechanism.

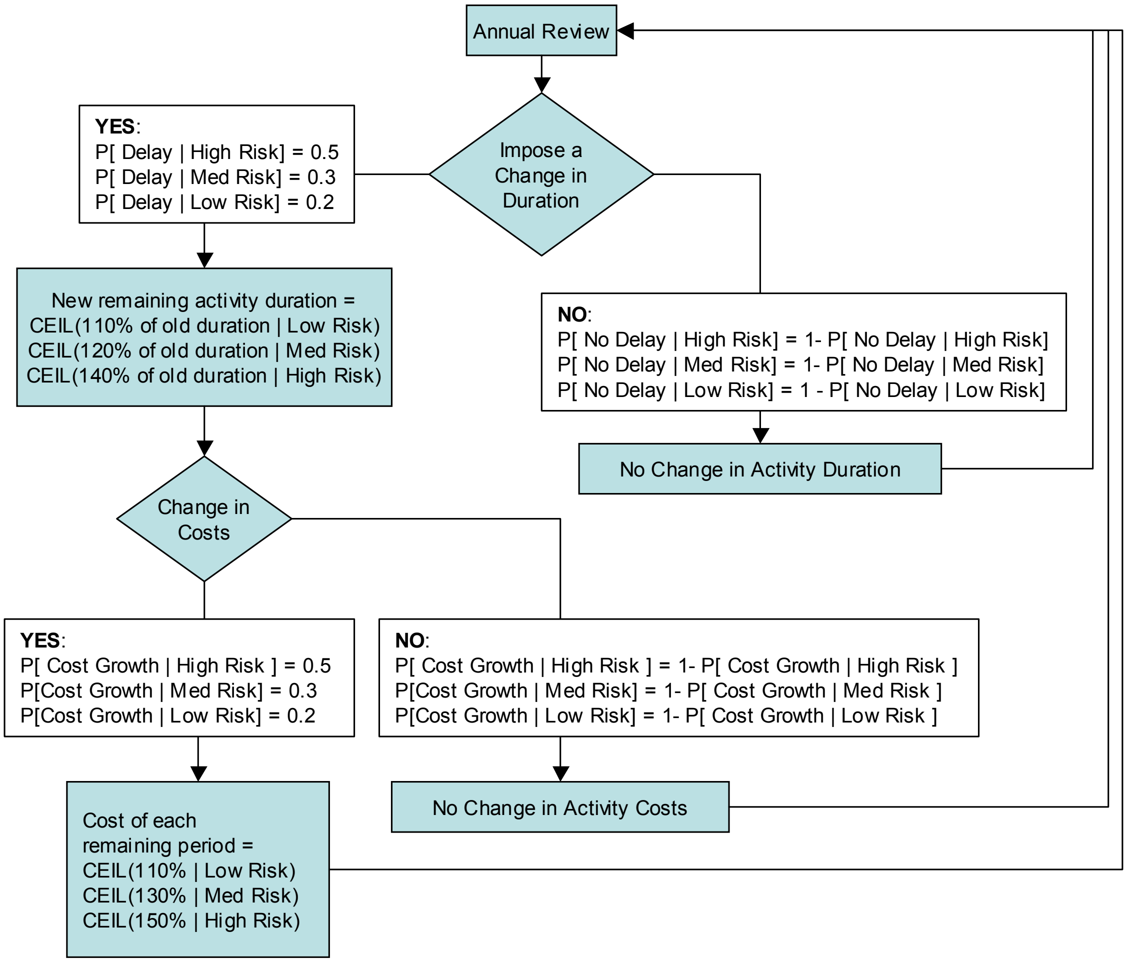

Figure 4. Annual Review Resampling Mechanism
A review of every underway task is conducted as depicted in this flow chart. Tasks may be delayed such that they are reviewed again later, and thus may be delayed further. No task may be delayed more than twice its initial planned maximum duration.

If the re-sampling extends the remaining project duration, the total cost is increased in proportion to the length of the extended duration. The new cost is re-spread over the longer duration using the Rayleigh distribution technique described earlier. Re-sampled task durations are then treated as deterministic.

The original maximum duration estimate for each task (d_{Max}) is equivalent to the maximum planning duration at the start of a project. In actuality, however, defense project tasks are often delayed beyond this point. In the optimization model a task can be

delayed up to twice its original planned maximum duration. This represents an absolute upper bound beyond which a task would not be delayed further as it would probably be cancelled.

If the ensuing project plan takes longer than originally planned due to budget shortfalls, a decision will need to be made whether to seek a supplemental appropriation, or to continue spending under the current budget. As one of the inputs, a vector of project budgets has been prepared for each possible year of project completion, which is presented in Chapter VI.

Annual reviews are repeated through the end of the planning horizon or project completion. The model does not make provision for the conditioning of cost or duration on any successor task as a result of a predecessor taking longer than planned. Such condition can be accommodated, but data on dependencies between tasks are not available.

The cumulative effect of annual reviews on project duration is shown in Figure 5 for a simple four-task project. Under scenario 3, which has no annual reviews, the project finishes in about three and a half years. Under scenario 4, which has annual reviews until the project is completed, the project finishes in about five years. At the first annual review one underway task is considered for cost and/or schedule growth, and is delayed by about one-quarter year. Tasks that have not started must be rescheduled to account for the later finish of the first task, which increases the projected duration of the project to almost four years. At the second annual review, the first task has finished and the second and third tasks are underway. Rescheduling decisions concerning these two underway tasks delay the project even further. These delays accumulate over the succession of annual reviews, which explains the longer project duration under scenario 4 vis-à-vis scenario 3. The effect of multiple annual reviews in this example is representative of schedule growth resulting from such factors as annual budget reallocations, contract disputes and defects with product performance.

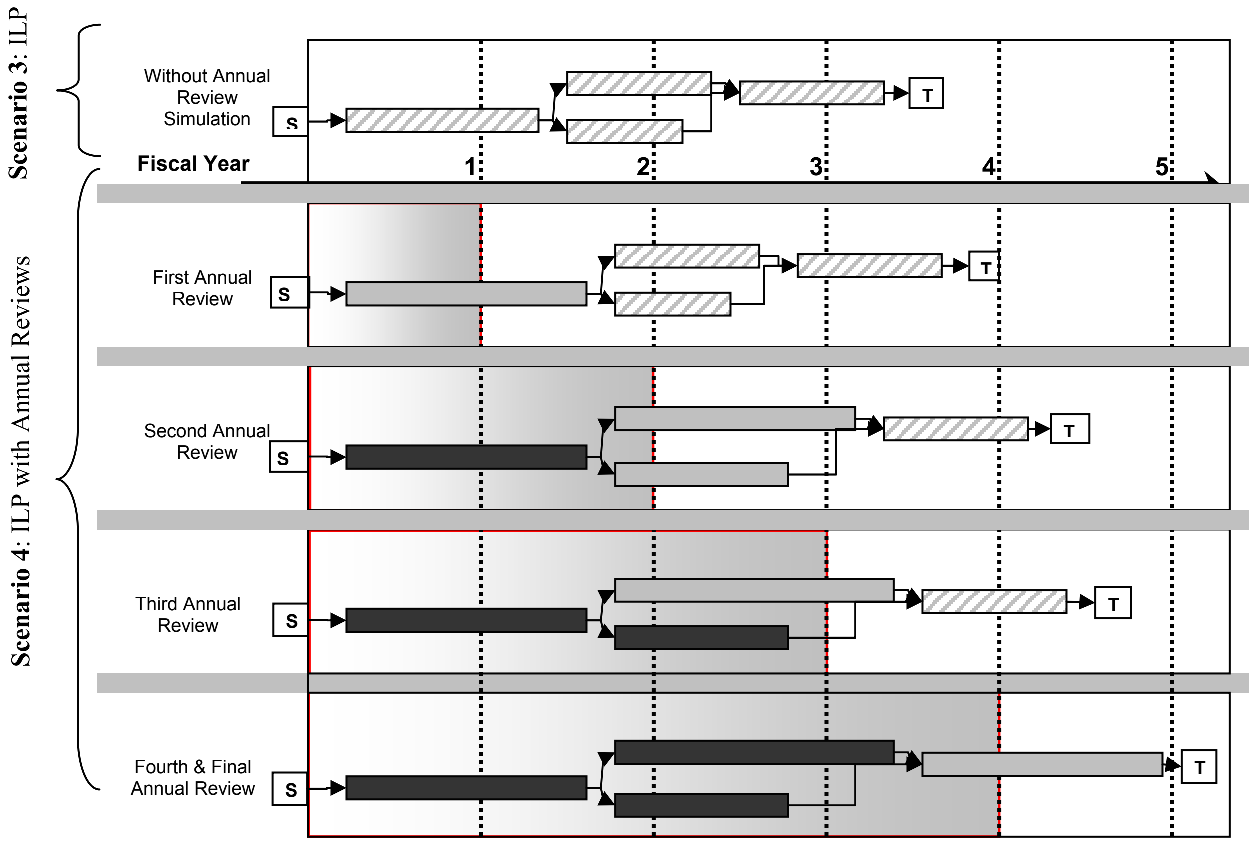

Figure 5. Abstraction of the Moving Window for the Simulated RCPSP with Annual Review

Start of the project is signified by $\boxed{S}$ and project end by $\boxed{T}$. Scenario 3 is the optimal deterministic schedule prior to any annual reviews. Scenario 4 uses the ILP to optimally reschedule projects after each annual review. Project end times are delayed as the reviews progress. Tasks marked with diagonal stripes are un-started so are not subject to any annual reviews. Gray tasks are underway at the time of the review, and may be subject to cost and/or schedule growth. Black tasks have finished and past scheduling decisions are fixed for all future annual reviews. The large shaded boxes show progress of annual reviews through time.

Schedule slippage of underway tasks expresses some of the uncertainty of real-world projects. As the project schedule is re-optimized in each succeeding annual review using different task costs, it is possible to obtain estimates of the probability distributions of outcomes such as project duration and cost. Competing schedule plans can then be compared to infer which has the best risk profile at any given time.

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V. EXPERIMENTAL RESULTS

Information obtained from our sponsor on tasks related to FCS acquisition is reported in Appendix B. Using this data, results are presented for the unconstrained schedule analysis described in Chapter III (scenarios 1 and 2), and for the resource-constrained schedule optimization described in Chapter IV (scenarios 3 and 4). The three schedule plans that are considered (baseline plan, alternate plan 1, and alternate plan 2) are described in Chapter I.

The aim is to identify differences between the three plans to produce a ranking based on completion time: a faster schedule is preferable to a slower one. Because much of the data on FCS are either classified or proprietary, the sponsor supplied hypothetical information, which is sufficient to demonstrate concepts described in the thesis. Where omissions existed, reasonable assumptions were made.

A. FCS INPUT DATA

In optimization scenarios 3 and 4, annual expenditures are constrained to fall within budget ranges. The ranges used in the experiment are based on a FCS project cost estimate of approximately \$20 billion developed from public source material obtained from the research sponsor. This cost estimate covers the SDD phase and early production of FCS, and is based on starting in 2003 and ending in 2012. Using the Rayleigh cost-spread formula presented in Chapter IV, this cost estimate is allocated on an annual basis from 2003 to the projected year of completion. A collection of annual budgets is called a "project budget". A separate project budget is required for each feasible completion year ranging from 2010 to 2016.

For each year of completion after 2012 the cost estimate is increased by 0.5 percent compounded annually. This inflation factor reflects, on a percentage basis, the estimated cost increase of \$4 billion to \$5 billion reported by Francis (2004) for a one-year delay in completion of FCS. Conversely, planning to accelerate the pace of work to complete the project one year early is assumed to require an increased budget of 0.2 percent (Lee, 1997).

In a given year the ILP is constrained by minimum and maximum budgets which are 20 percent and 105 percent of the planned annual budgets respectively. The interval between these limits is called the "budget band" for that year. Tabulated budget bands corresponding to project completion years from 2010 to 2019 are shown in Part B of Appendix B. The ILP must select exactly one project budget to use. Violation of the budget band is permitted but is penalized by the ILP. These bands are chosen to be more restrictive with respect to over-expenditure than under-expenditure to reflect realistic budgetary conditions.

B. RESULTS OF THE UNCONSTRAINED STOCHASTIC SCHEDULE ANALYSIS

In the unconstrained schedule analysis the task durations are sampled from truncated three-parameter Weibull distributions, and an unconstrained reaching algorithm is used to search the schedule for the longest path, which is equivalent to the finish time of the last task. Results for the unconstrained reaching algorithm without simulation are shown in Table 3.

The project completion time is recorded and the simulation repeated for 60,000 iterations. Because individual project tasks have random durations, the project finish time is itself a random variable. Based on the simulation results, a distribution function for the last task finish time was estimated, and is shown in Figure 6.

Schedule Plan	Project Duration without Schedule Simulation (months)	Schedule Delay Compared to Army Plan (%)	Estimated Finish Month / Year
Baseline Plan	118	0	Sep 2012
Alternate Plan 1	116	-2	Jul 2012
Alternate Plan 2	129	9	Sep 2013

Table 3. Scenario 1: Unconstrained Project Completion Times without Simulation.

Project durations are measured from 1 Jan 2003.

The plan having a distribution that is most concentrated on lower values of completion time is the most desirable. Clearly, alternate plan 1 emerges as the preferred schedule. The baseline plan has the lowest probability of successful completion at any given time compared to the other plans.

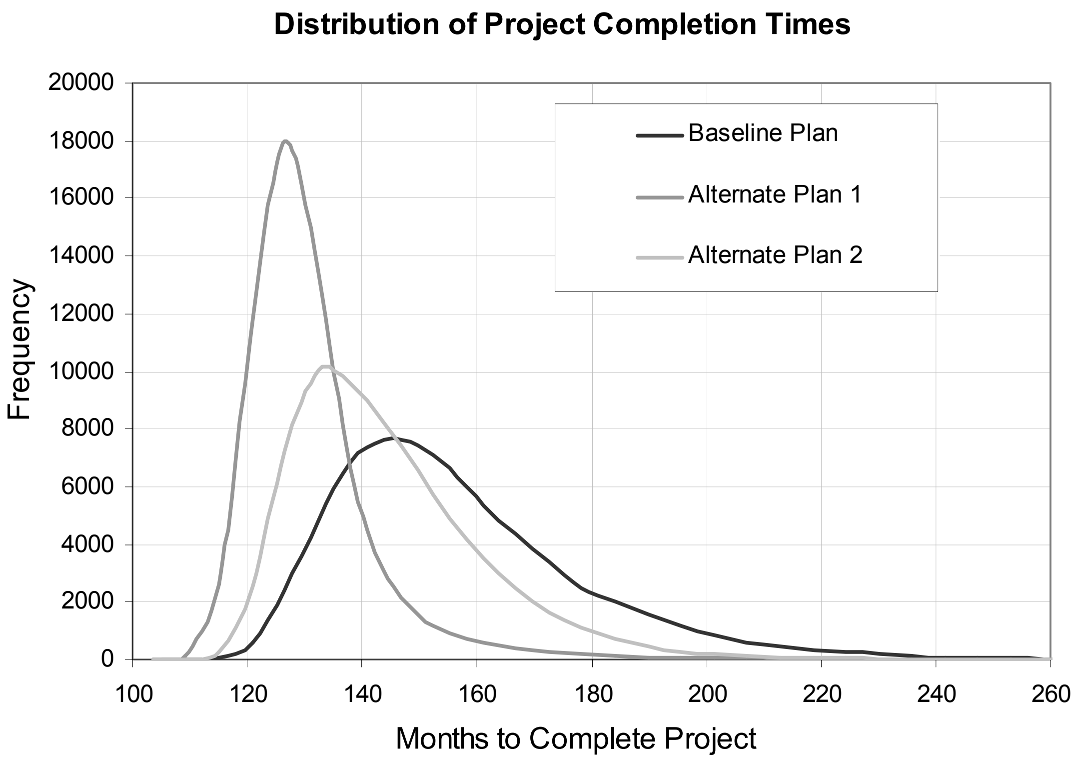

Figure 6. Scenario 2: Distribution of Finish Times from Unconstrained Reaching Algorithm with Simulation

One of the key assumptions that differentiate the plans is that the same task common to all three plans can be allocated different risk levels. For example, in the baseline plan "system integration" and "system testing" are high-risk tasks because immature technologies must be concurrently developed and integrated. If, however, the risky technologies are first matured prior to system integration as in alternate plan 1, then integration and testing tasks proceed at a reduced risk level compared to the baseline plan.

Simulation results are sensitive to changes in such assumptions, and can lead to results that at first seem counter-intuitive. A task with a long initial duration but low risk may actually finish sooner is the simulation than an initially shorter task that has high schedule risk. As was discussed in Chapter II, it is for precisely this reason that PERT estimates of project completion times are often optimistic.

Under current assumptions, alternate plan 1, which matures high-risk tasks before starting development on other components, is the preferred plan. Even though the risk mitigation tasks delay the start of overall system development, the gains in reduced integration and testing more than compensate for the early delays. Table 4 summarizes the results for the unconstrained reaching algorithm with schedule simulation.

Schedule Plan	Project Duration with Schedule Simulation (months)	Schedule Delay Compared to Army plan (%)	Estimated Finish Month / Year
Baseline Plan	150	27	Nov 2014
Alternate Plan 1	126	7	Mar 2013
Alternate Plan 2	139	18	Sep 2013

Table 4. Scenario 2: Unconstrained Project Completion Times with Simulation.
Project durations are measured from 1 Jan 2003. Project durations are medians of the distributions of finish times shown in Figure 6.

Project durations shown are the medians of the distributions shown in Figure 6. The median is a good measure of spread, as 50 percent of observations fall above and below it. Importantly, the median is not affected by extreme outliers to the same extent as the mean. This makes it an appropriate point estimate of likely project duration.

C. RESULTS OF THE BUDGET-CONSTRAINED DETERMINISTIC OPTIMIZATION ANALYSIS

The optimization model finds the most-compressed schedule that satisfies the overall project budget selected by the optimal project completion year. Penalized cumulative elastic budget constraints on the objective function allow for over-expenditure in any particular annual project budget provided there is a compensating under-expenditure in a later period.

The ILP balances the cost of taking longer to finish the project while strictly observing budget constraints, with penalties incurred by adopting a shorter schedule but exceeding annual budgets. Typically the latter option is favored by the ILP. As a result, the project expenditure profile does not follow that projected by the Rayleigh distribution of the budget.

In the annual review simulation, the costs and durations of tasks underway at each annual review are subject to growth. This exogenously changes the data, and the ILP must be re-solved in order to optimally re-schedule future tasks. As start time and duration decisions for completed tasks are already fixed, and start times for underway tasks are already fixed, re-scheduling future tasks is the only significant source of flexibility. As the program nears completion, and the number of future tasks nears zero, flexibility is nearly lost and the program must carry on as best as possible given past scheduling decisions.

1. Scenario 3: Deterministic Model without Annual Review Simulation

The Army had planned to reach a key FCS completion milestone (fielding a unit of action) by the end of September 2012. Results of applying the optimization model without the annual review simulation are shown in Table 5. These completion times are the fastest possible budget-constrained finish times as no random task delays are imposed. These effectively form a lower bound on project completion time. Each of the three plans induces a different expenditure profile, as discussed separately below.

Schedule Plan	Project Duration (months)	Schedule Delay Compared to Army Plan (%)	Estimated Finish Month / Year
Baseline Plan	118	0	Sep 2012
Alternate Plan 1	116	-2	Jul 2012
Alternate Plan 2	130	10	Sep 2013

Table 5. Scenario 3: Constrained Project Completion Times without Annual Review Simulation

Project durations are measured from 1 Jan 2003.

a. Results for the Baseline Plan without Annual Review Simulation

Expenditure for the baseline plan is shown in Figure 7. In the first two years, spending exceeds the budget, so a penalty would be have been imposed by the ILP. This is balanced by under-spending in later years.

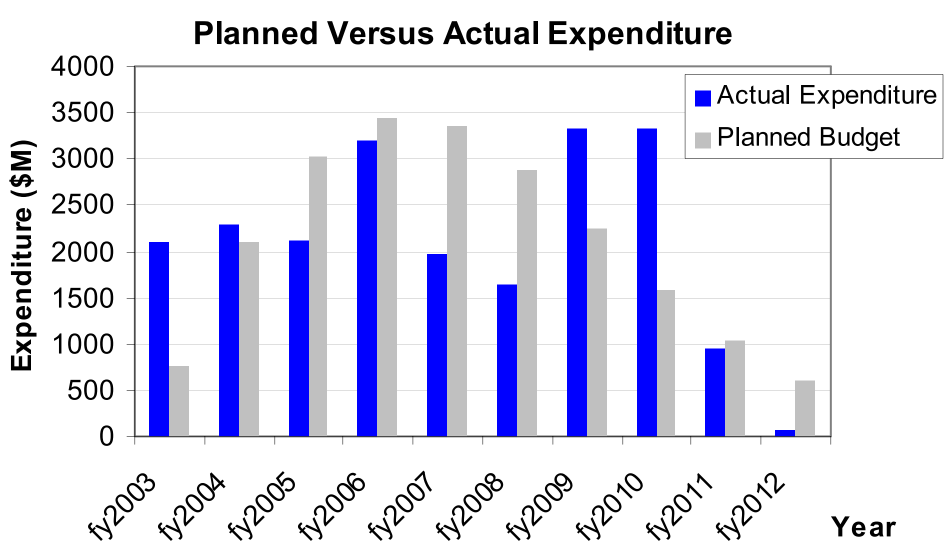

Figure 7. Project Expenditure without Annual Review Simulation for Baseline Plan

The ILP must balance the benefit of undertaking many tasks simultaneously by a penalized over-expenditure with an approach that does not incur penalties but takes longer to complete the project. Cumulative expenditure is shown in Figure 8.

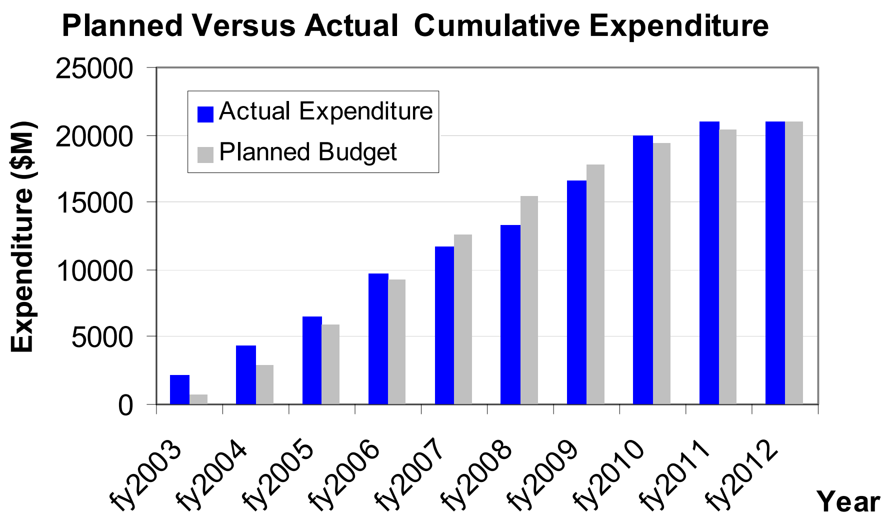

Figure 8. Cumulative Project Expenditure without Annual Review Simulation for Baseline Plan

Figures 7 and 8 show that spending under the baseline plan dips below budget to build a reserve of cash for the two very expensive years of 2010 – 2011, which mark the start of pre-production tasks. The Army aimed to complete system development and demonstration phase by the end of 2010, which is not achieved under the baseline plan.

b. Results for Alternate Plan 1 without Annual Review Simulation

Expenditure under alternate plan 1, shown in Figure 9, is initially high due to the simultaneous risk mitigation tasks that must be completed prior to starting main developmental tasks. Significant under-spending through the middle of the project, obvious in Figure 10, builds a reserve to pay for the relatively expensive pre-production tasks at the end of the project The Army aimed to complete system development and demonstration phase by the end of 2010, which is not achieved under alternate plan 1.

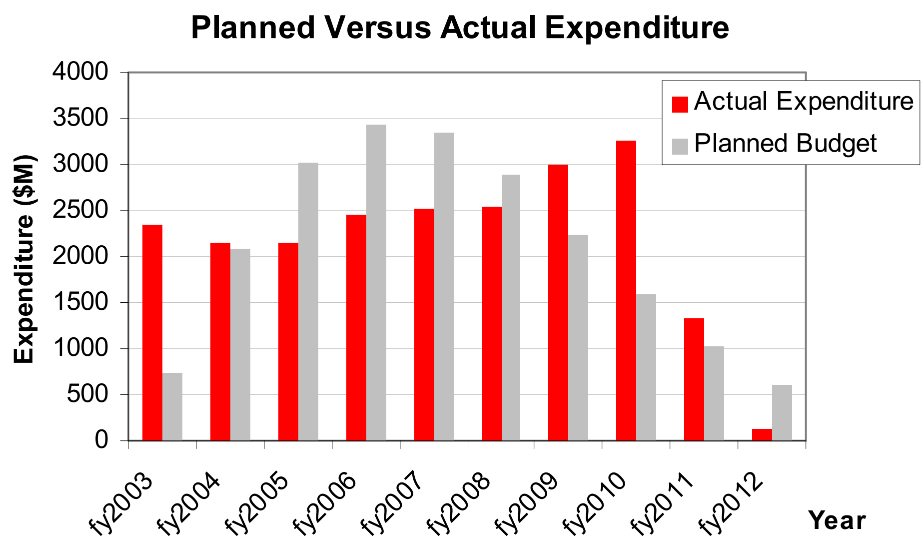

Figure 9. Project Expenditure without Annual Review Simulation for Alternate Plan 1

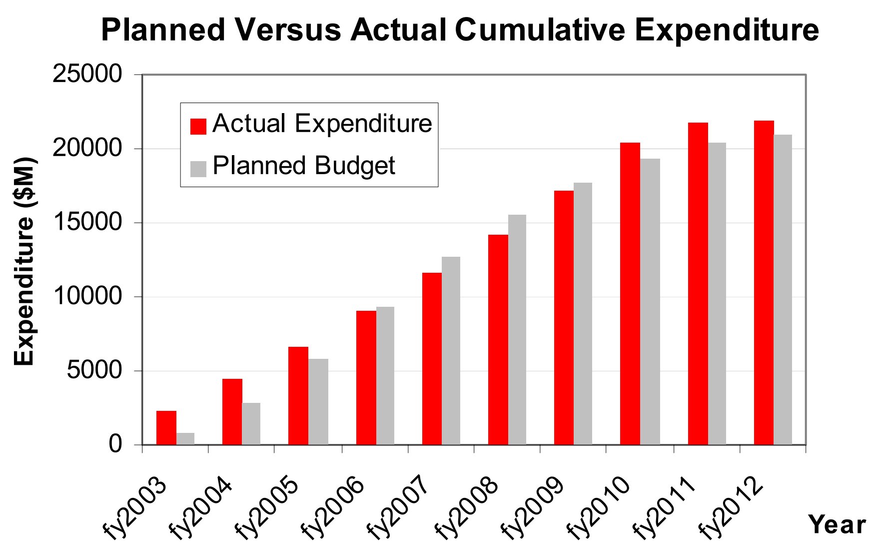

Figure 10. Cumulative Project Expenditure without Annual Review Simulation for Alternate Plan 1

c. Results for Alternate Plan 2 without Annual Review Simulation

Expenditure under alternate plan 2, shown in Figure 11, is initially high as expensive software development tasks must be completed prior to starting other development tasks. Once the C4ISR tasks are completed there is a significant period of under-spending, obvious in Figure 12, in order to build a large reserve of cash to pay for

the pre-production tasks. The spike in expenditure for pre-production is most pronounced in this plan. The Army aimed to complete system development and demonstration phase by the end of 2010, which is not achieved under alternate plan 2. Because this plan has the longest duration of the three plans considered, it is the least desirable at this stage.

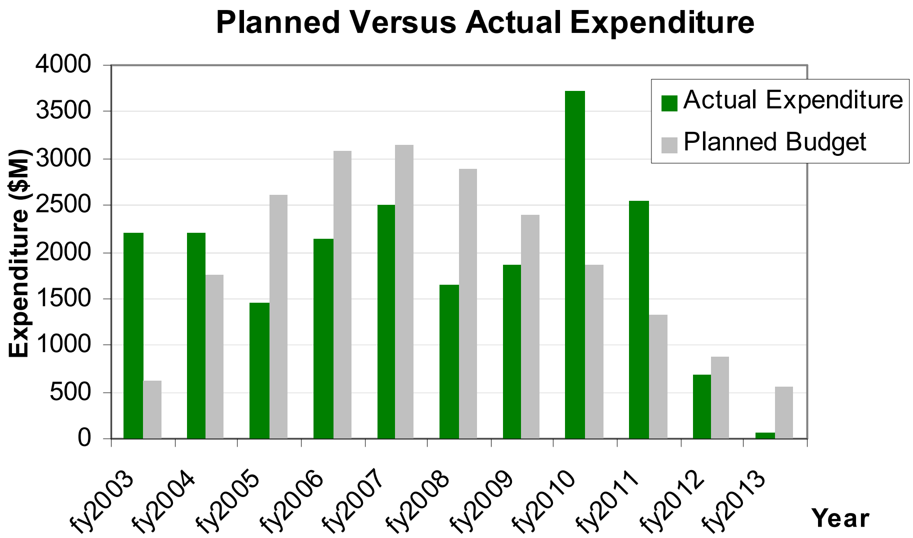

Figure 11. Project Expenditure without Annual Review Simulation for Alternate Plan 2

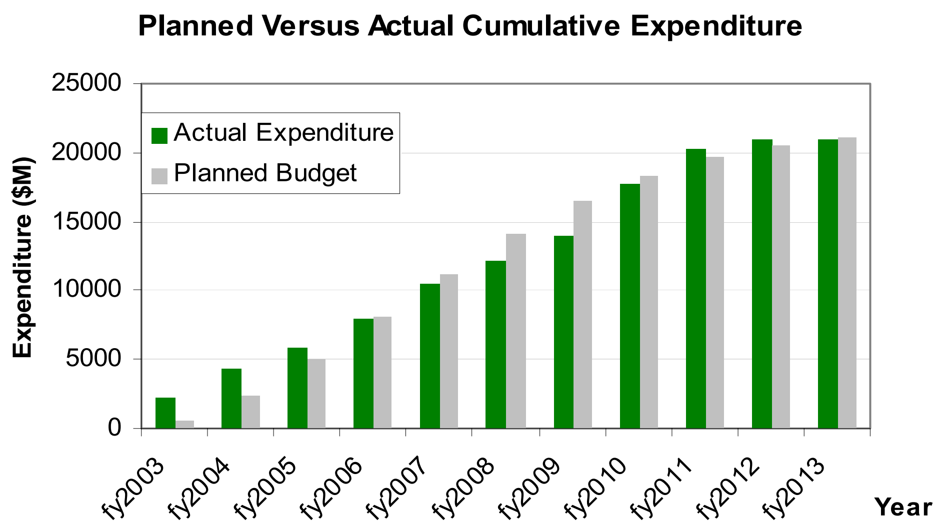

Figure 12. Cumulative Project Expenditure without Annual Review Simulation for Alternate Plan 2

Plotting the three expenditure profiles together in Figure 13 and 14 reveals a similar pattern. Many tasks are started simultaneously, and as a result there is an initial period of over-expenditure.

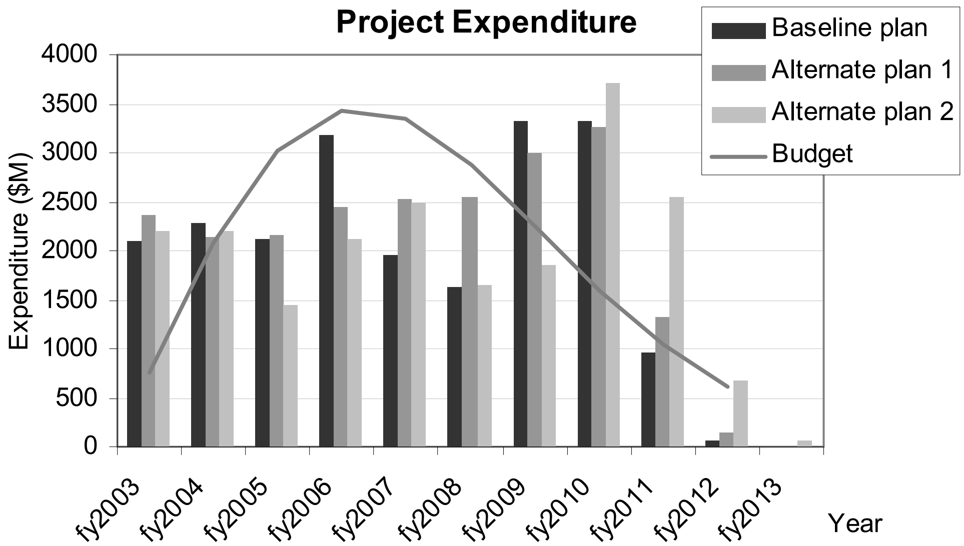

Figure 13. Project Expenditure without Annual Review Simulation for All Schedule Plans

The annual budget plotted with a line is based on project completion in 2012 with costs allocated to years using the Rayleigh distribution. Note that early years over-spend so later years can be afforded.

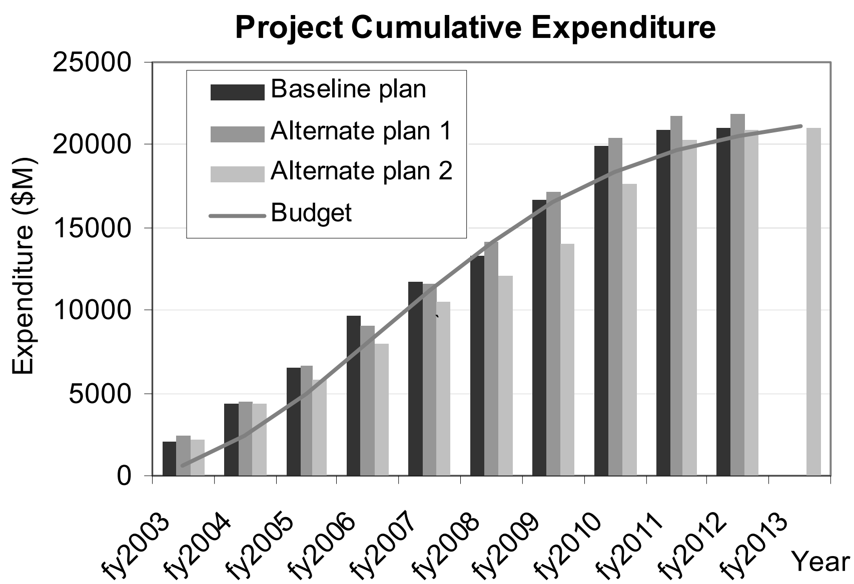

Figure 14. Cumulative Project Expenditure without Annual Review Simulation for All Schedule Plans

As the project progresses, expenditure drops below budget to build a reserve needed to fund the expensive pre-production tasks that occur at the end of the project. Alternate plan 1 entails the greatest cost, although it has the shortest duration.

Initial over-expenditure of this nature is generally taken as a warning sign in project management. Experience suggests that early over-expenditure does not imply that a program will subsequently under-spend in order to observe overall budget constraints. This is because early overspending is often invested in infrastructure and personnel that will be available ahead of their actual need, resulting in an increased budget requirement later in the project (Cooper, 2003). Combined with the difficulty of scheduling tasks to comply with the project budget, use of the Rayleigh distribution to allocate the budget over the project duration may be undesirable. Other distributions, including those suggested by Jarvis (2001) may be more appropriate for this purpose.

2. Scenario 4: Deterministic Model with Annual Review Simulation

When the annual task reviews with random delays on underway tasks are introduced, overall project duration increases. A plot of simulated project schedule completion times estimated by the annual review simulation is shown in Figure 15.

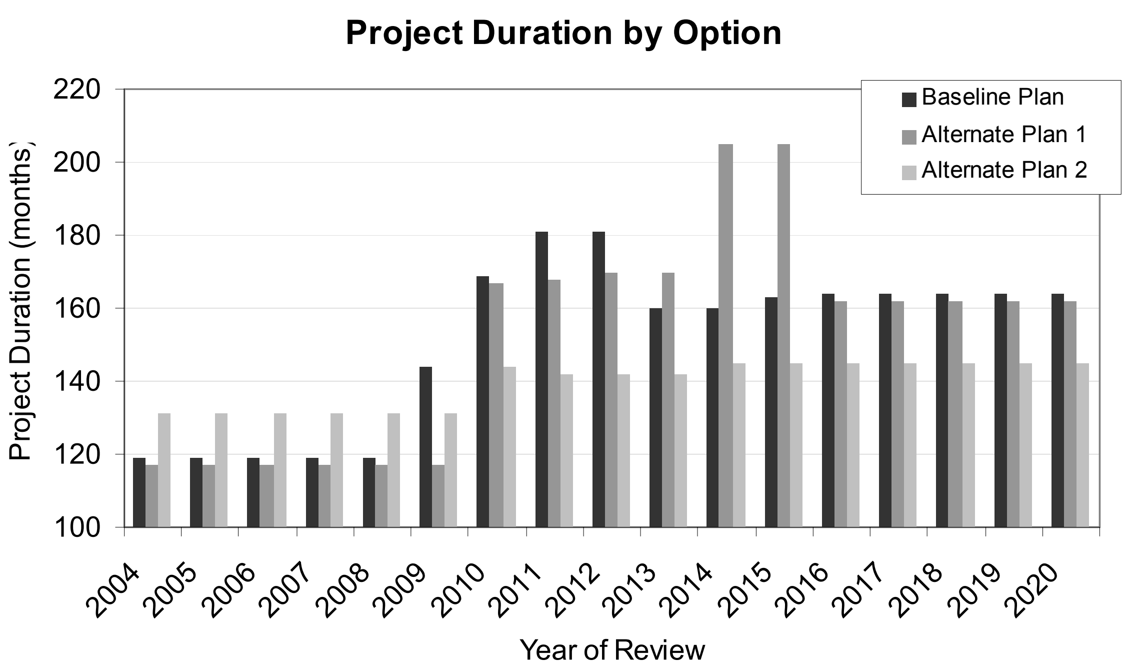

Figure 15. Project Completion Times with Annual Reviews

The estimated project durations are summarized in Table 6 for the three schedule plans considered.

Schedule Plan	Project Duration (months)	Delay compared to Army plan (%)	Estimated finish month / year
Baseline Plan	164	39	Jul 2016
Alternate Plan 1	162	37	May 2016
Alternate Plan 2	145	23	Jan 2015

Table 6. Scenario 4: Constrained Project Completion Times with Annual Review Simulation

As would be expected, project completion times are increased as tasks are delayed, but then project finish times decrease which initially seems counter-intuitive. This is due to an increase in available budget in the year of the decrease. The ILP can select a larger budget at the end of each fiscal year to increase the budget for following years. If the budget is increased, more tasks can be performed concurrently than what would previously have been the case. This results in a shorter project completion time. This is made clearer by examining a budget decomposition that reveals the variable nature of project completion times.

3. Budget Decomposition

As schedules for each of the three plans are determined, the ILP can select a larger project budget to either allow more tasks to take place concurrently, or to cover an extended project duration. A vector of possible budgets has been developed for every feasible year of project completion. The schedule from the annual review simulation may spend in accordance with more than one of the vector of possible annual budgets

prepared at the project outset. Figure 16 shows the budget decomposition for the solution to the baseline plan. At any point in time, exactly one of the budgets is used, but upon completion of the annual review simulation it is possible to construct the single budget that was used during the annual review simulation. The solid line indicates the budget that the baseline plan actually used.

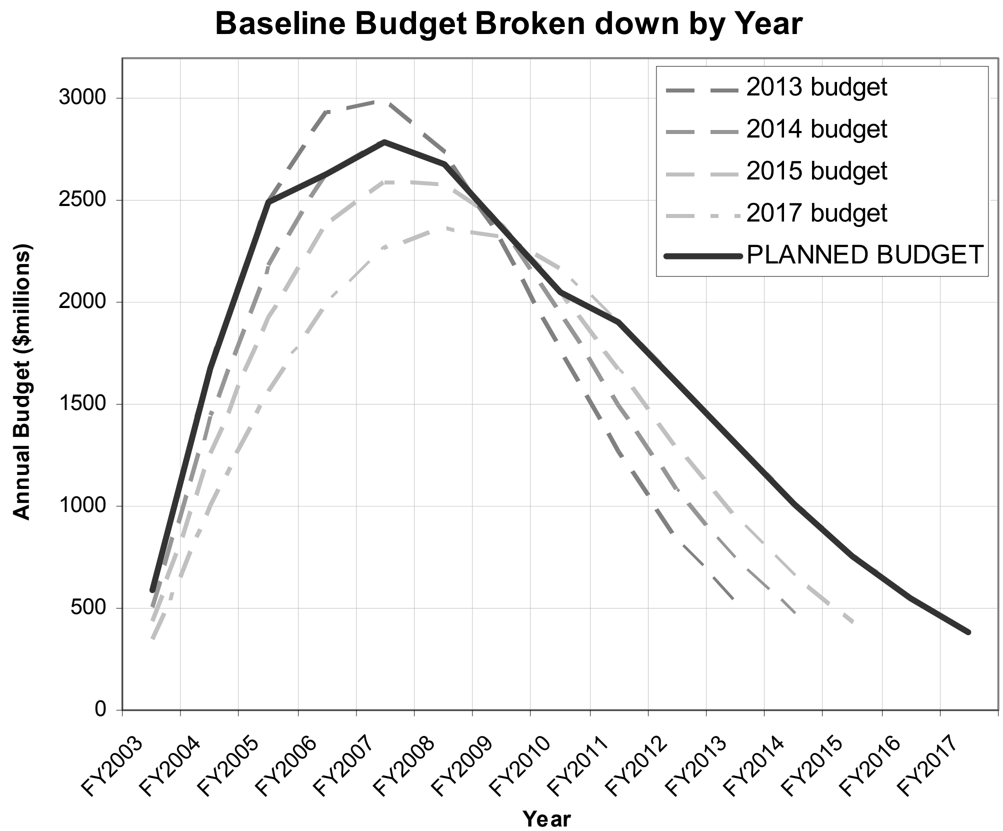

Figure 16. Baseline Plan Budget Broken down by Project Completion Year

Each dashed line represents a project budget indexed by completion year; spread using the method based on the Rayleigh distribution. The solid line represents the decisions made by the ILP to spend against larger and longer budgets.

To be more explicit, the ILP used the budget for an estimated completion in FY2013 from FY2003 to FY2005. In FY2006, the program was no longer able to complete by FY2013, and so the FY2014 budget was selected. In FY2010 the program was delayed further the FY2015 budget was required. Further delays were imposed

during this year and in FY2011, the estimated completion time was extended to FY2017. The budget for the FY2017 completion time was sufficient to complete the project.

As the baseline plan budget switches between several of the project budgets indexed by completion year, the total budget used exceeds the totals of any of the project budgets. This is clear in the plot of cumulative budgets shown in Figure 17. Each dashed line represents one of the project budgets that were selected during the annual review simulation.

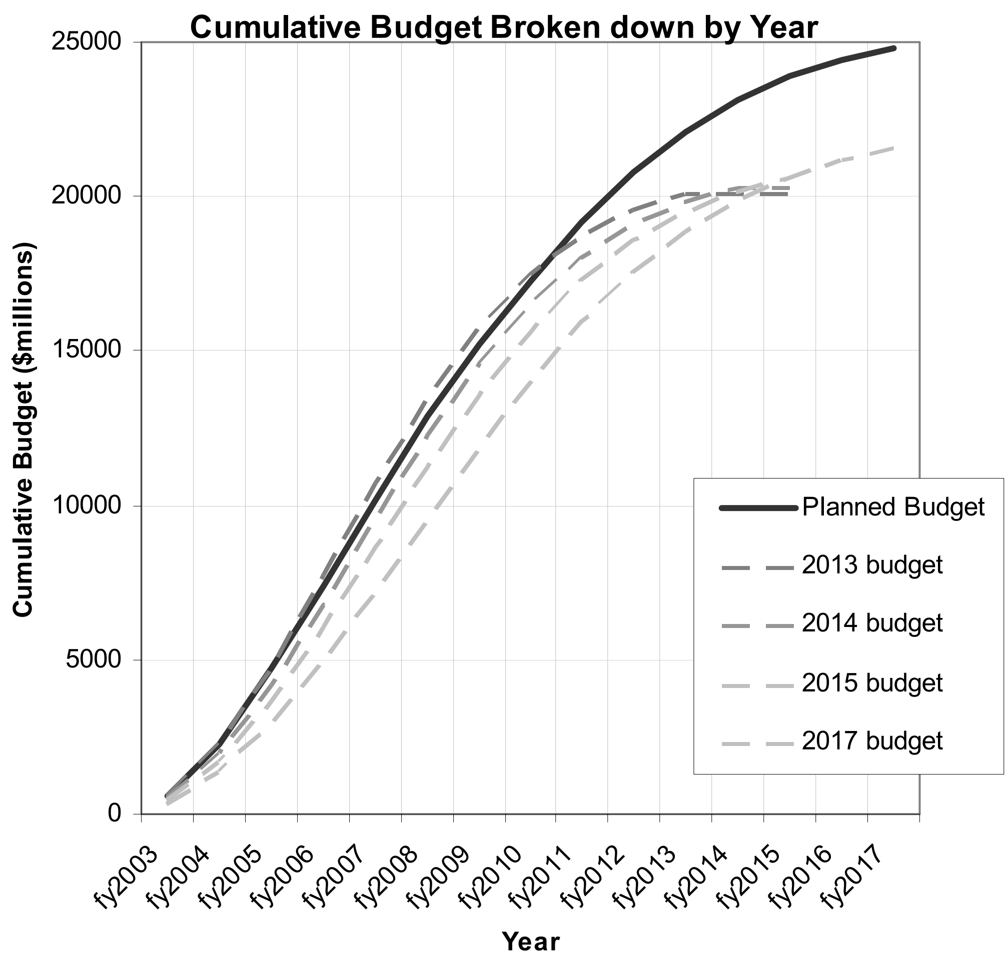

Figure 17. Baseline Plan Cumulative Budget Broken down by Project Completion Year

Each dashed line represents a project budget indexed by completion year; spread using the method based on the Rayleigh distribution. The solid line represents the decisions made by the ILP to spend against larger and longer budgets.

a. Results for the Baseline Plan with Annual Review Simulation

The results for each of the three plans are briefly reviewed to identify similarities. Expenditure for the baseline plan is shown in Figure 18 and cumulative expenditure in Figure 19.

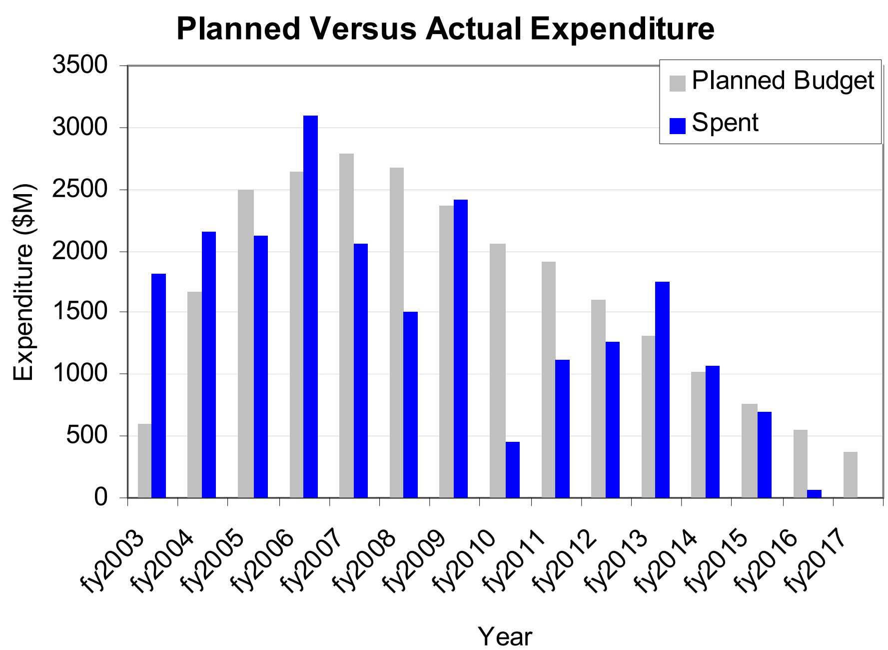

Figure 18. Annual Project Expenditure with Annual Review Simulation for Baseline Plan

Early over-expenditure is balanced by later under-expenditure. This mix provides the optimal balance of early completion by over-spending early and the penalties imposed from such spending. The budget is larger than needed to cover development tasks, but the larger budget is required to cover the longer project duration.

The baseline plan achieves a relatively quick completion in 2013, but requires a larger budget that stretches to 2017. Over-expenditure in early periods is balanced by under-expenditure (or no expenditure) in later periods.

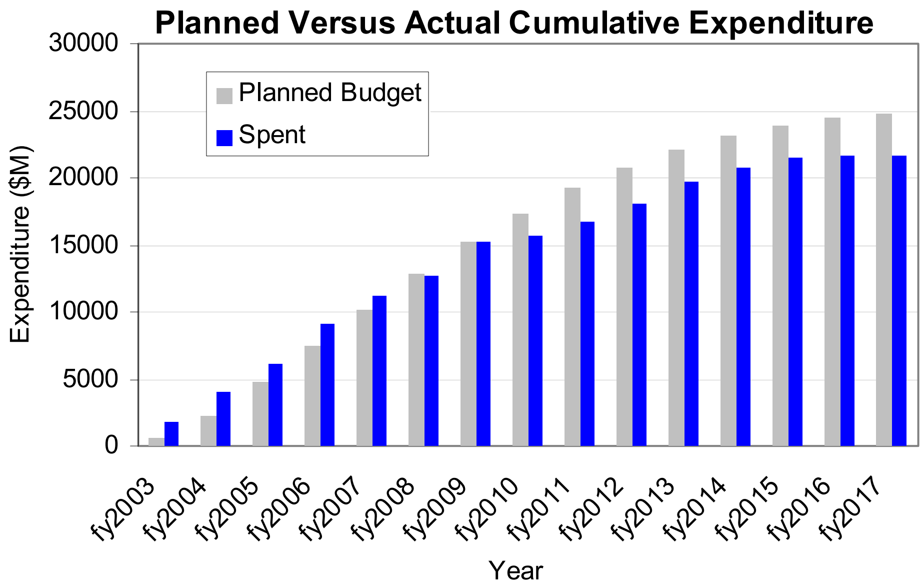

Figure 19. Cumulative Project Expenditure with Annual Review Simulation for Baseline Plan

In Figure 20, the results with annual review simulation are compared to those without simulation. An additional $605 million is spent as a result of the annual review simulation.

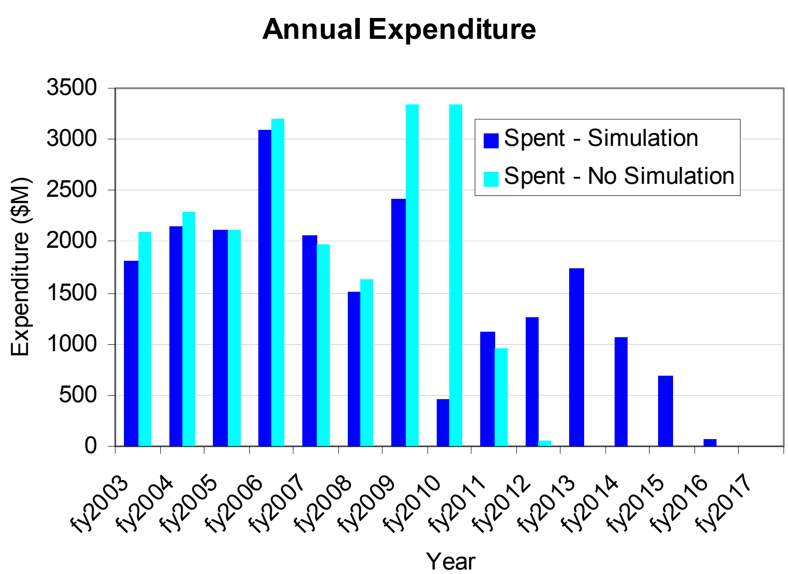

Figure 20. Project Expenditure with and without Annual Review Simulation for Baseline Plan

To cover the longer duration in the annual review simulation a longer budgetary horizon is required. The budget required in the annual review simulation is $3.8 billion more than in the deterministic case. The different budgets are shown in Figure 21.

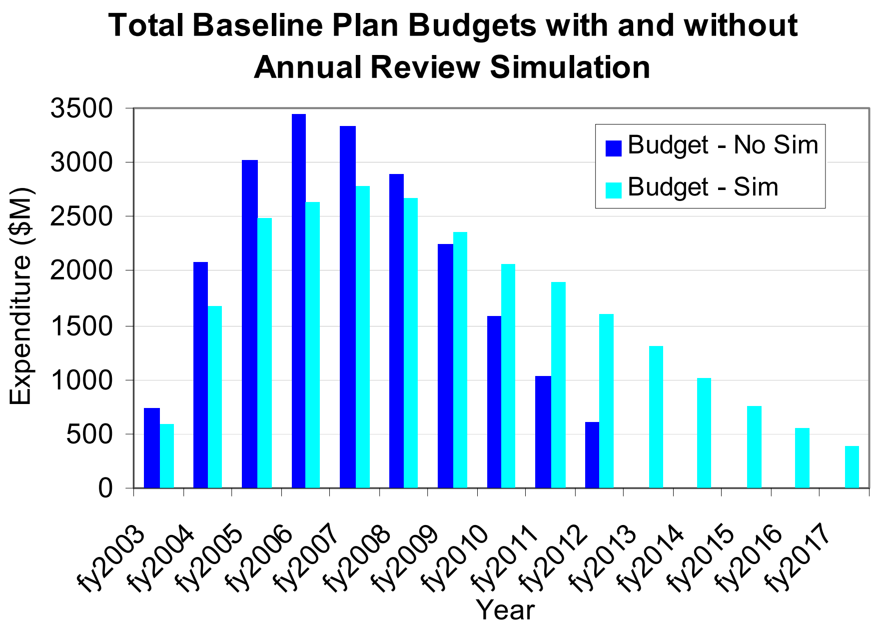

Figure 21. Project Budgets with and without Annual Review Simulation for Baseline Plan

b. Results for Alternate Plan 1 with Annual Review Simulation

Alternate plan 1 requires an initially high expenditure because many risk-mitigation tasks are started simultaneously. After the riskier tasks are completed, other developmental tasks are started. Due to the high levels of early expenditure, Figure 22 shows that there are long periods of under-expenditure in order to finish below the overall project budget. Figure 23 shows cumulative expenditure.

For alternate plan 1, Figure 24 shows the contrast in expenditure under schedule optimization with (scenario 4) and without (scenario 3) an annual review simulation. The introduction of an annual review results in an additional expenditure of $150 million due to decisions made to delay tasks.

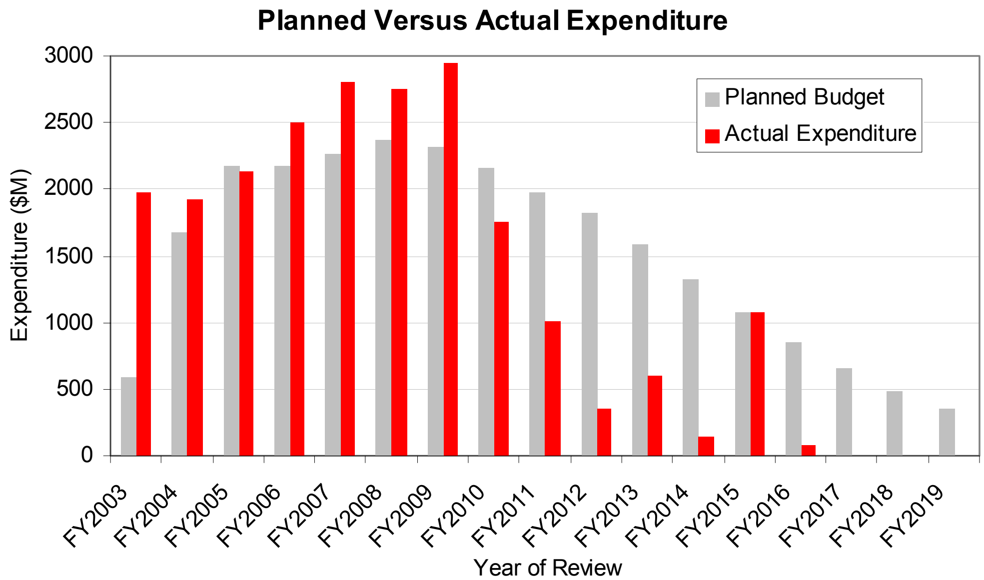

Figure 22. Project Expenditure with Annual Review Simulation for Alternate Plan 1

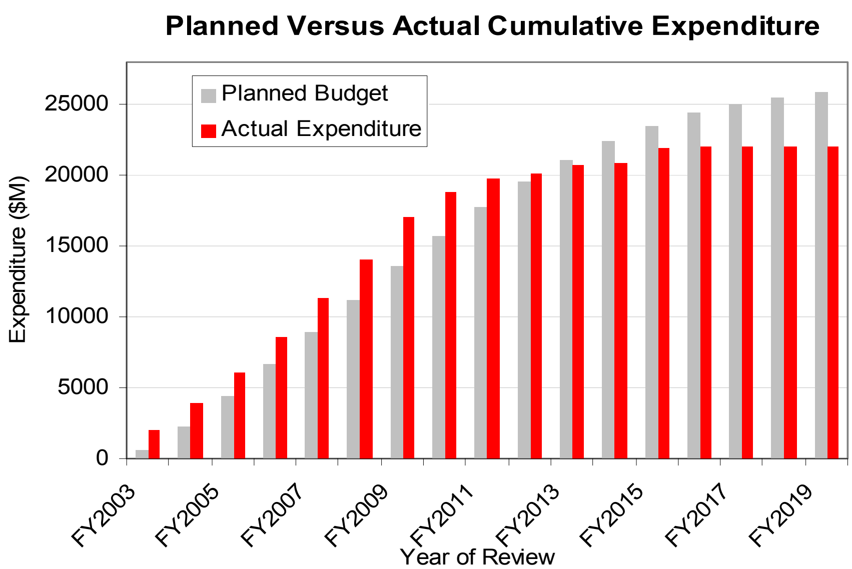

Figure 23. Cumulative Project Expenditure with Annual Review Simulation for Alternate Plan 1

The budget is larger than that needed to cover development tasks, but is required to cover the longer project duration.

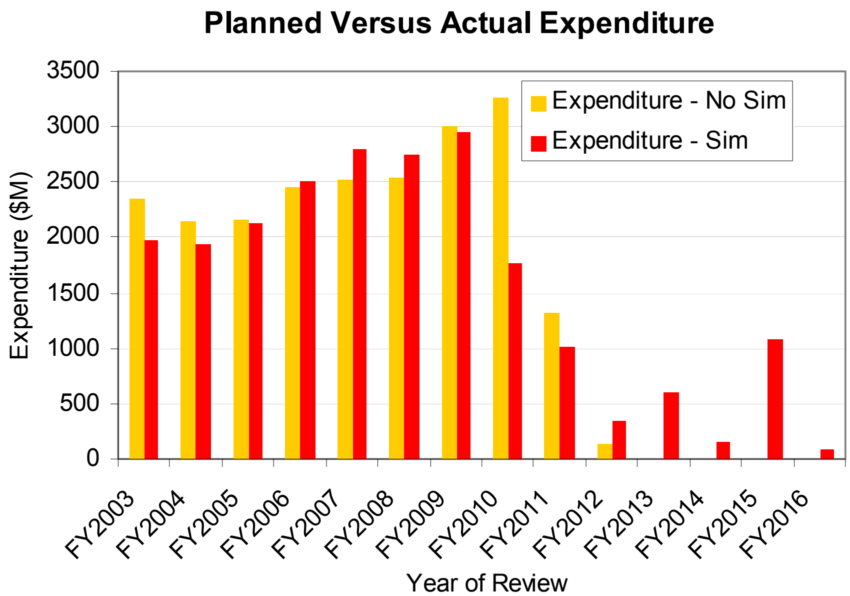

Figure 24. Project Expenditure with and without Annual Review Simulation for Alternate Plan 1

Alternate plan 1 requires a larger budget with the annual review simulation than without the simulation to cover the longer schedule duration, as shown in Figure 25. The cumulative difference between the two budgets is $4.8 billion, which is the largest difference among the three plans considered.

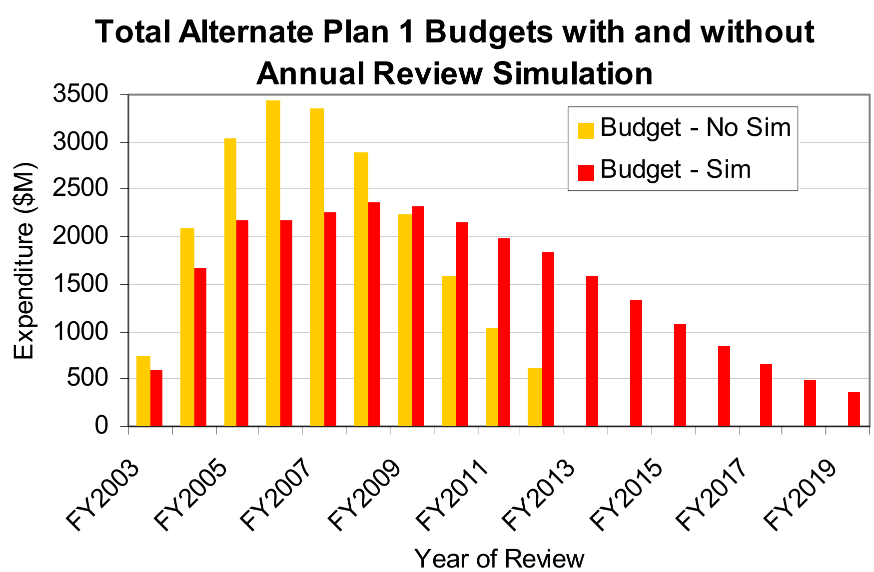

Figure 25. Project Budgets with and without Annual Review Simulation for Alternate Plan 1

c. Results for Alternate Plan 2 with Annual Review Simulation

Alternate plan 2 follows the pattern of the baseline plan and alternate plan 1, namely early over-expenditure, followed by under-expenditure in preparation for a spike in expenditure at the end of the project. The pattern, however, is less pronounced than in the other plans. Due to the initially high expenditure, Figure 26 shows that there must be periods of under-expenditure in order to finish below the overall project budget. Figure 27 shows the cumulative expenditure for alternate plan 2.

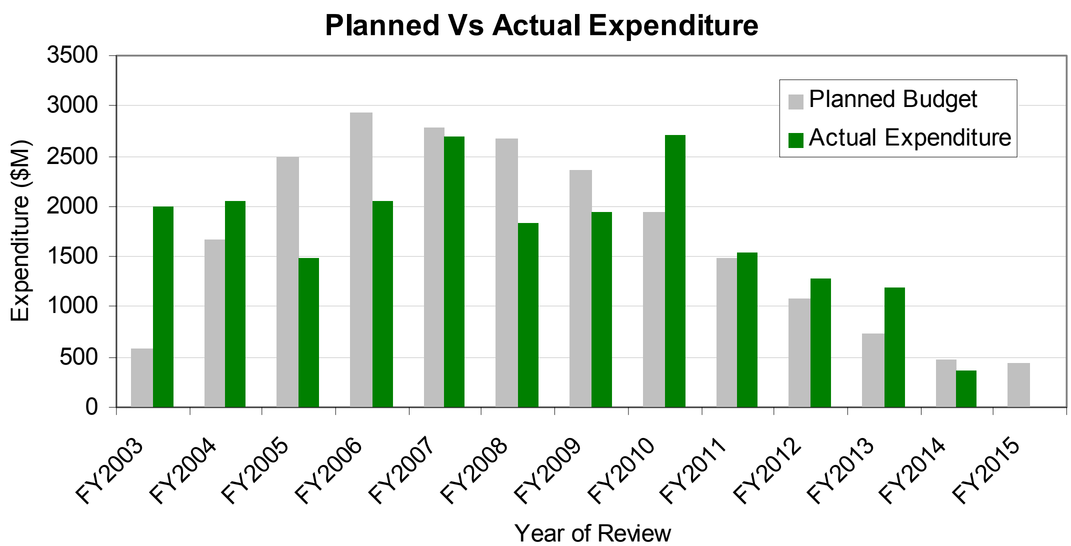

Figure 26. Project Expenditure with Annual Review Simulation for Alternate Plan 2

For alternate plan 2, Figure 28 shows the contrast in expenditure under schedule optimization with (scenario 4) and without (scenario 3) an annual review simulation. The introduction of an annual review results in an additional expenditure of $168 million due to decisions made to delay tasks.

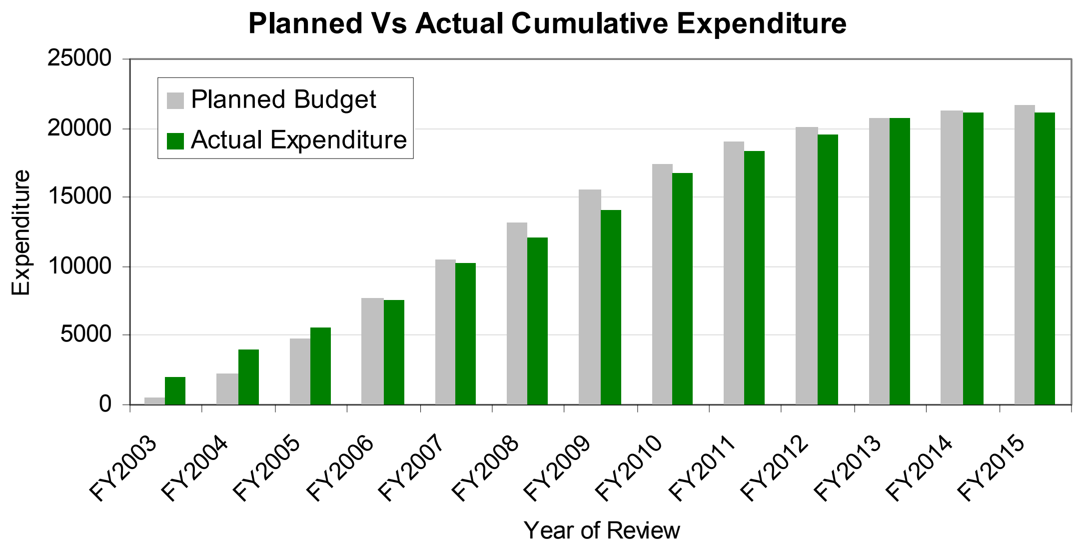

Figure 27. Cumulative Project Expenditure with Annual Review Simulation for Alternate Plan 2

Alternate plan 2 requires a larger budget with the annual review simulation than without the simulation to cover the longer schedule duration, as shown in Figure 29. The cumulative difference between the two budgets is $564 million, which is the smallest difference among the three plans.

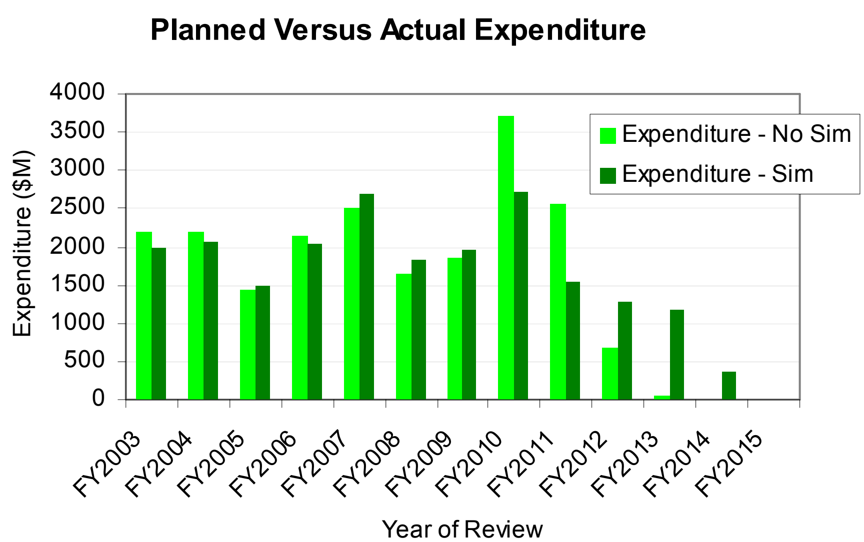

Figure 28. Project Expenditure with and without Annual Review Simulation for Alternate Plan 2

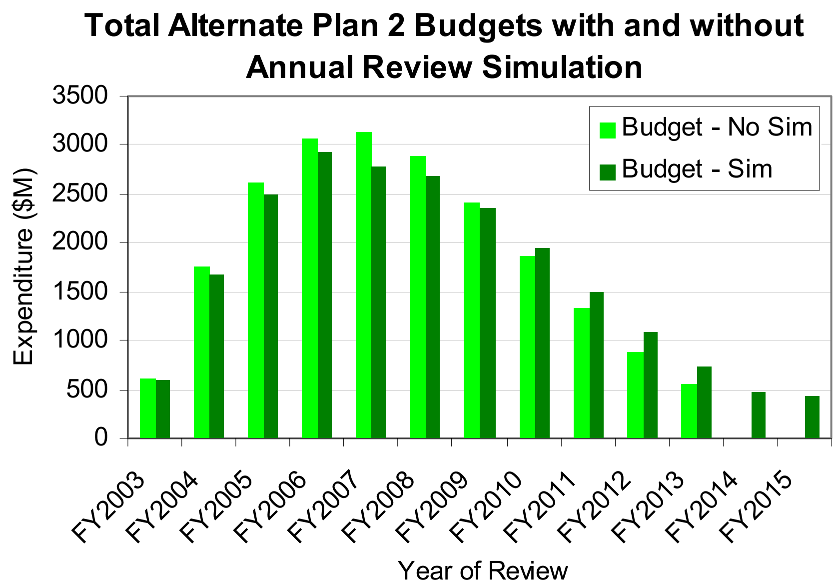

Figure 29. Project Budgets with and without Annual Review Simulation for Alternate Plan 2

Building the C4ISR system first results in a schedule which is the most robust to variability in activity durations.

4. Plan Comparisons

Figure 30 shows expenditure profiles for all three plans analyzed under Scenario 4. Cumulative expenditures are shown in Figure 31. Fiscal years where overspending occurs are common to all three plans.

It is clear that all plans greatly overspend in the early periods, and rise to a general peak in the 2010-2011 period. At that time, the project is nearing completion and the expensive early production tasks are nearly complete. Alternate plan 2 is cheaper than the other plans as it finishes years earlier and requires a smaller budget to cover all tasks.

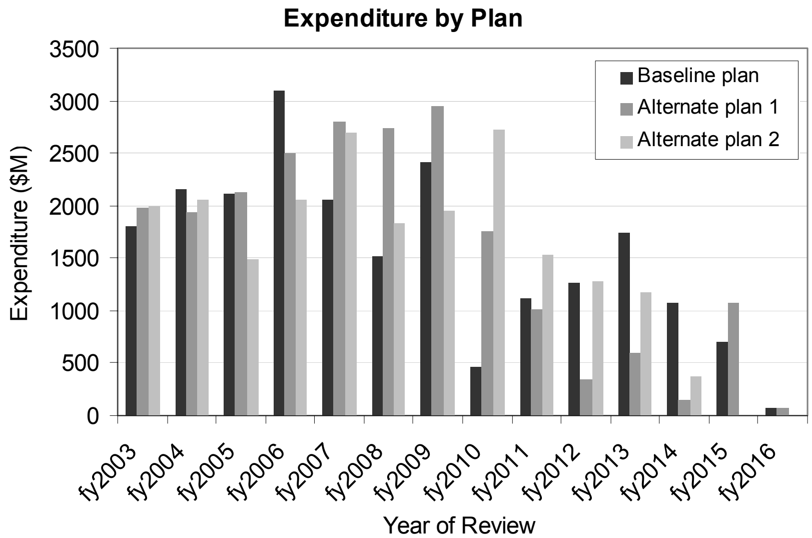

Figure 30. Annual Project Expenditure by Schedule Plan

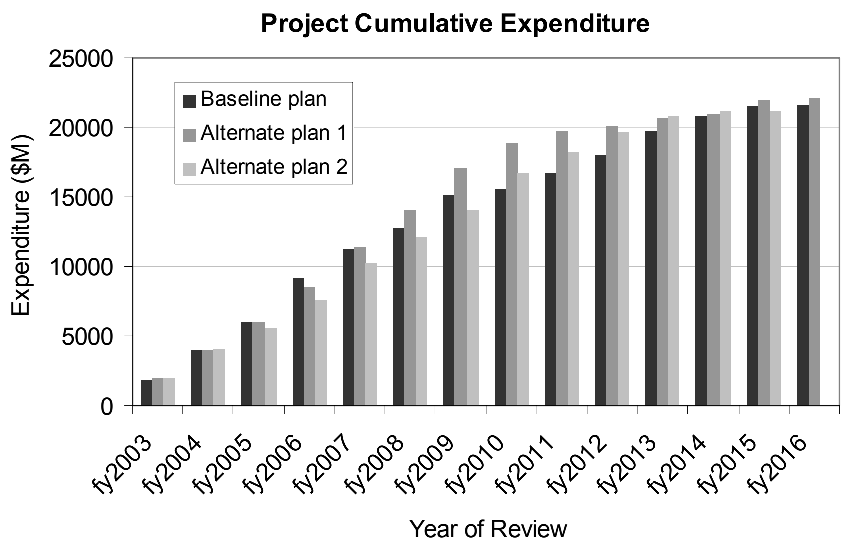

Figure 31. Cumulative Project Expenditure by Schedule Plan

Strong expenditure by alternate plan 1 is most pronounced in the FY2008 – FY2011 period when it is clearly greater than the other plans.

5. Solution Times and Convergence

ILP solution times are longer than expected due to the difficulty of scheduling multiple task costs within an overall project budget, both of which are modeled by Rayleigh distributions. The problem is complicated by the annual budget constraint, the magnitude and duration of which is controlled by a decision variable in the ILP.

The CPLEX solver (ILOG, 2004) was used to solve the ILP in the analysis described above. The solver was unable to find a feasible solution unless it was provided one as a starting point. The pre-generation of a feasible starting solution complicated the GAMS implementation. Furthermore, several CPLEX features failed including integer cuts and CPLEX bogged down in problem pre-processing. Most of the advanced CPLEX options for cut generation and root node heuristics needed to be disabled. All ILP solutions were found using branch-and-bound variable searching.

In the first few stages of the annual review simulation the branch-and-bound tree is vast, and the optimal solution is sometimes not found. The gap between the best solution found thus far and the theoretical best solution can be significant. As the annual review simulation progresses, the ILP has fewer choices to make which reduces the problem size and the size of the gap generally decreases. Near the end of the annual review simulation the ILP converges to the optimal solution and the gap between the best solution found thus far and the optimal solution is closed to zero.

The gap between the lower and upper bound is not monotonically non-decreasing. As an increase of project budget is a relaxation in the ILP, and an imposed increase in project duration is a restriction, the result is not obvious. The complex interaction of relaxations and restrictions in this optimization formulation leads to gaps that both increase and decrease as each annual review is conducted, although the magnitude of changes decrease as the annual review simulation progresses.

6. Number of Simulation Realizations

Results presented in this chapter are for a single realization of the ILP annual review simulation. Additional realizations would be required before the mean project duration could be expected to converge to a steady state value. The number of

realizations may need to exceed 40 in order to invoke the Central Limit Theorem. This requires an investment of computational resources beyond the scope of the thesis, but the results presented in this thesis are representative of what can be expected. Moreover, as the data used in this thesis are nominal (actual FCS project information is classified beyond the clearance of the author), a significant computational investment to determine steady state conditions may not be warranted without real data in hand.

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VI. CONCLUSIONS

A. CONCLUSIONS

Schedule plans for the System Development and Demonstration Phase of the Future Combat Systems have been examined in this thesis using an integer linear program to optimize the completion time of the last task within annual and project budget constraints. Alternate plan 2, which requires that the C4ISR infrastructure be built prior to other components, displays the most robust schedule in the transition from Scenario 1 to 4. Other schedules are delayed to greater extents and require larger budgets than under alternate plan 2. An overview of the results is shown in Table 7.

	Program Delay Relative to Current Army Plan (%)			
	Without Budget Constraints		With Budget Constraints	
Schedule Plan	**Scenario 1** Without Schedule Simulation	**Scenario 2** With Schedule simulation	**Scenario 3** Without Annual Reviews	**Scenario 4** With Annual Reviews
Baseline Plan: Proceed with current Army planned schedule.	0	27	10	39
Alternate Plan 1: Mitigate high risk technologies prior to other tasks.	-2	7	8	37
Alternate Plan 2: Develop the C4ISR system prior to other systems.	9	18	21	23

Table 7. Overview of Percentage Schedule Delay by Plan and by Scenario

Figures indicate the percentage delay that a program experiences compared to what the FCS program office has forecast. For example, the baseline plan finish time under the fourth scenario experiences is delayed 39% compared to the FCS Program Executive Office estimate.

If there are no budget constraints, mitigating high risk technology first is preferred both with and without schedule simulation. When budget constraints are added, this option and the current Army project plan are both attractive. Annual review simulation of the constrained optimization reveals that the third plan is the least vulnerable to delay. Given the high levels of schedule risk that exist in the FCS program, alternate plan 2 is the recommended plan because it is least affected by the effects of uncertainty.

Solution times for the ILP are longer than expected. The CPLEX solver used required a pre-processed feasible solution to be provided as a starting point, and this complicated the implementation. CPLEX had difficulty finding cuts to reduce problem size, and bogged down in pre-processing. All solutions to the ILP are found using branch and bound variable search techniques. As the annual review simulation progresses, the solution found by the solver converges towards the optimal solution.

B. FUTURE AREAS OF STUDY

The analytical approach presented in this thesis is generally applicable to the scheduling of major defense acquisition projects. The budget-constrained optimization model requires that the decision maker supply two parameters that characterize the durations of tasks within a project, namely an estimated most likely duration and a categorical schedule risk assessment. Although this simplifies the task for the decision maker, care must be exercised in selection of these parameters to ensure that useful results are obtained from the analysis. It would be useful to examine the efficacy of current guidelines for parameter selection, and to propose refinements that better match what has been observed in practice. Assumptions of cost growth of the overall project schedule as a function of longer completion times may need to be reviewed.

There are many extensions that can be made to the optimization model itself. Recovering the critical path at the end of each annual review would reveal which tasks are more commonly on the critical path. Statistics on the frequency with which tasks fall on the critical path could also reveal potential flaws with the project design. This could be an aid to efficiently directing management effort, capacity planning and priorities for future procurement.

More sophisticated elastic penalties could be developed to better reflect the decisions made by real project managers. A questionable aspect of this model is its tendency to produce scheduling solutions that over-spend early in the project, compensated by under-spending during the mid-project phase. In practice an early over-expenditure typically results in over-expenditure throughout the project.

Nonetheless, the tendency to schedule activities very early in the project and in doing so incurring large penalties implies that this is better than waiting for the money to become available according to the Rayleigh spread of the project budget. Evidently, the Rayleigh budget spread does not schedule enough money early in the project. Schedule optimization prefers to exceed the budget early and repay the shortfall later in the project. It allows these violations of the Rayleigh budget spread albeit by imposing penalties. As a result, expenditure profiles do not resemble the Rayleigh budget spread.

In the optimization individual task expenditures are spread over their durations using Rayleigh distributions, and the overall project budget is also spread using a Rayleigh distribution. It is difficult to impose these constraints simultaneously without allowing for violations. This suggests the use of alternate models for spreading project budgets, which may achieve developmental goals with fewer penalties, less cost, and earlier completion times than the options presented here.

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LIST OF REFERENCES

Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1993). *Network Flows: Theory, Algorithms and Applications*, Prentice Hall, Englewood Cliffs.

Babbar, M.M., Tintner, G., and Heady, E.O. (1955). "Programming with Consideration of Variations in Input Coefficients", *Journal of Farm Economics*, May, **37**, No. 2, pp. 333-341.

Bachmann, P.G.H. (1894). *Die Analytische Zahlentheorie*, Teubner, Leipzig. (unsighted).

Bowman, E.H. (1958). "The Schedule Sequencing Problem", *Operations Research*, **7**, pp. 621-624.

Brady, E. (2003). *Future Combat Systems,* Modeling and Simulation Conference, Project Executive Office Ground Combat Systems, U.S. Army, 30 Jan 2003. Accessed 14 Jan 2004 via the World Wide Web at http://www.boeing.com/defense-space/ic/fcs/bia/modsim_030130.html .

Brook, A., Kendrick, D., Meeraus, A., and Raman, R. (1998). *GAMS: A User's Guide,* The Scientific Press, Washington DC.

Cooper, W. (2003). Cost Analysis Improvement Group, Program Analysis and Evaluation, Office of the Secretary of Defense, Various interviews, Dec 2003.

Demeulemeester, E. and Herroelen, W. (2002). *Project Scheduling: A Research Handbook*, Volume 49 of the International Series in Operations Research and Management Science, Kluwer Academic Publishing, Boston.

Dodin, B. (1984). "Bounding the Project Completion Time Distribution in PERT Networks", *Operations Research*, **33**, pp. 862-881.

Francis, P.L. (2004). *Defense Acquisitions: The Army's Future Combat Systems' Features, Risks and Alternatives*, GAO-04-635T, testimony before the Subcommittee on Tactical Air and Land Forces, Committee on Armed Services, U.S. House of Representatives, April 2004.

Fulkerson, D.R. (1962). "Expected Critical Path Lengths in PERT Networks", *Operations Research*, **10**, pp. 808-817.

Gutjahr, W.J., Stauss, C., and Wagner, E. (2000). "A Stochastic Branch and Bound Approach to Activity Crashing in Project Management", *INFORMS Journal on Computing*, **12**, No. 2, pp. 125-135.

Herroelen, W. and Leus, R. (2004). "Robust and Reactive Scheduling: A Review and Classification of Procedures", *International Journal of Production Research*, **42**, No. 8 (15 April), pp. 1599-1620.

Herroelen, W., Reyck, B.D., and Demeulemeester, E. (1998). "Resource-Constrained Project Scheduling: A Survey of Recent Developments", *Computers & Operations Research,* **25,** No. 4, pp. 279-302.

Igelmund, G. and Radermacher, F.J. (1983). "Pre-selective Strategies for the Optimization of Stochastic Project Networks under Resource Constraints", *Networks*, **13**, pp. 1-28.

ILOG (2004). ILOG Optimization Suite White Paper, Accessed 13 Jun 2004 via the World Wide Web at http://www.ilog.com/products/optimization/papers.cfm.

Jarvis, W. (2001). "New Approaches to Phasing of Resources", In *Proceedings of the 34th Department of Defense Cost Analysis Symposium*, Washington DC. Accessed 17 Jun 2004 via the World Wide Web at http://www.ra.pae.osd.mil/adodcas .

Kelly, J.E. (1961). "Critical Path Planning and Scheduling: Mathematical Basis", *Operations Research*, **9**, pp. 296-320.

Kelly, J.E. (1963). "The Critical Path Method: Resource Planning and Scheduling", in Muth J.F. and Thompson G.L. (Eds.), *Industrial Scheduling*, Prentice Hall, Englewood Cliffs.

Lee, D. (1997). *The Cost Analyst's Companion*, Logistics Management Institute, McLean, VA.

Malcolm, D.G., Roseboom, J.H., Clark, C.E., and Fazar, W. (1959). "Application of a Technique for Research and Development Program Evaluation", *Operations Research*, **7**, pp. 646-669.

Miller, S. (2003). "Simulation Based Schedule Analysis", In *Proceedings of the 36th Department of Defense Cost Analysis Symposium*, Washington DC. Accessed 12 Dec 2003 via the World Wide Web at http://www.ra.pae.osd.mil/adodcas/index2.htm.

Pritsker, A.B., Watters, L.J., and Wolfe, P.M. (1969). "Multi-Project Scheduling with Limited Resources, A Zero-One Programming Approach", *Management Science*, **16**, pp. 93-108.

Sengupta, J.K., Tintner, G., and Morrison, B. (1963). "Stochastic Linear Programming with Applications to Economic Models", *Economica*, **30**, pp. 262-275.

Senju, S. and Toyoda, Y. (1968). "An Approach to Linear Programming with 0-1 Variables", *Management Science*, **15**, No. 4, pp. 196-207.

Tintner, G. (1955). "Stochastic Linear Programming with Applications to Agricultural Economics", In Antosiewicz, H.A., *Second Symposium on Linear Programming*, National Bureau of Standards, Washington, Jan 27-29 1955.

Tintner, G. (1960). "A Note on Stochastic Linear Programming", *Econometrica*, **28**, No. 2, pp. 490-495.

U.S. General Accounting Office (2003). GAO-03-1010R *Issues facing the Army's Future Combat Systems Program*, Washington DC, Aug 2003.

Ullman, J. (1975). "NP-Complete Scheduling Problems", *Journal of Computer Systems Science*, **10**, No. 3 (June), pp. 384–393.

Weisstein, E.W. (2004). "Reaching Algorithm", *MathWorld*, National Science Foundation. Accessed 11 Jan 2004 via the World Wide Web at http://mathworld.wolfram.com/ReachingAlgorithm.html.

Weist, J.D. (1964). "Some Properties of Schedules for Large Projects with Limited Resources", *Operations Research*, **12**, pp. 395-418.

Welch, L.D. (2003). *Objective Force and Future Combat Systems Independent Assessment Panel Report*, Institute for Defense Analysis, Alexandria, VA.

Wynne, M.W. (2003). *Operation of the Defense Acquisition System*, DoD Instruction 5000.2, Office of the Principal Deputy Under Secretary of Defense for Acquisition, Technology and Logistics, U.S. Department of Defense, May 12 2003.

Yang, B., Geunes, J., and O'Brien, W.J. (2001). *Resource-Constrained Project Scheduling: Past Work and New Directions*, Research Report 2001-6, Department of Industrial and Systems Engineering, University of Florida.

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APPENDIX A

A. MANNED GROUND VEHICLES

Representative Picture	Description
	**Prototype** Demonstrate increased mobility, survivability technologies and designs.
	**LOS/BLOS** Vehicle Combat vehicle with 105-120mm cannon with Line of Sight (LOS) and Beyond LOS (BLOS) capability. Also included is a Self Protection Weapon.
	**NLOS Cannon** Vehicle Combat vehicle with 120-155mm cannon with Non Line of Sight (NLOS) capability. This system incorporates technologies that include CARGO rounds and smart sub-munitions, and Fire and Forget Seeker technology. Also included is a Self Protection Weapon.
	**NLOS** Mortar Vehicle Combat vehicle with 120mm mortar gun with NLOS capability. Also included is a Self Protection Weapon.
	**Missiles Vehicle** Combat vehicle carries missiles-in-a-box configuration that minimizes reloading time and effort. The Missile system provides BLOS precision guided missiles and loitering munitions. Also included is a Self Protection Weapon.

	**Armored Personnel Carrier** Transports a full 9-man infantry squad including their associated gear and 2-soldier crew. Also included is a Self Protection Weapon.
	**Control Vehicle (CV)** Provides 4-soldier workstation, 1 driver and 1 commander. Used for control of UGV's and UAV's. Also included is a Self Protection Weapon.
	**Command and Control (C2) Vehicle** Provides 4 soldier workstation, 1 driver and 1 commander. Provides the connection among the Force and communication on the move. Also included is a Self Protection Weapon.
	**Re-supply Vehicle** General-purpose vehicle with embedded semi-autonomy provides operation as a follower. Crew size consists of 1 driver and 1 commander. Also included is a Self Protection Weapon.
	**Reconnaissance, Surveillance, and Target Acquisition (RSTA)** Vehicle Integrates RSTA suite of 5-meter mast, thermal imagers (Long-Wave Infrared (LWIR) and Medium-Wave Infrared (MWIR)), day/night television (TV) camera, 10 km+ laser range finder, Ka band radar, and 360 deg. all elevation azimuth. Provides 2 soldier workstation, 1 driver and 1 commander. Also included is a Self Protection Weapon.
	**155 mm Re-supply Vehicle** Provides automated re-supply to the 155-mm NLOS vehicle; re-supplied with palletized ammunition: Crew size consists of 1 driver and 1 commander. Also included is a Self Protection Weapon.

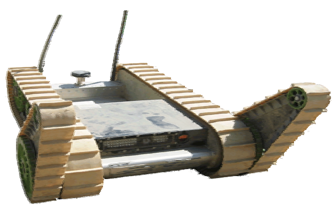
	**Recovery Vehicle** Provides towing and recovery assistance. Crew size consists of 1 driver and 1 commander. Also included is a Self Protection Weapon.
	**Medical Vehicle** Vehicle provides evacuation and/or medical treatment. Provides 1 injured station, 1 driver and 1 commander.
Image unavailable	**Bridge Vehicle** Equipped to lay bridge.
Image unavailable	**Mobility/Countermobility** Vehicle Equipped to breach and lay minefields, can be operated semi-autonomously; mission package includes a scraper, flail, and Mongoose. Vehicle has semi-autonomous capability. Also included is a Self Protection Weapon.
	**Small Unmanned Air Vehicles (SUAV) Launcher Carrier** Vehicle Transports a pod of 32 SUAV's and the launching system. Crew size consists of 1 driver and 1 commander.

B. UNMANNED AERIAL VEHICLES (UAV)

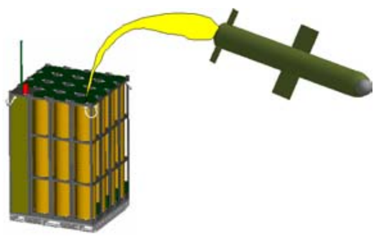
Representative Picture	Description
	The Future Combat Systems will develop four classes of UAVs. Class 1 will be a platoon-class small aircraft. Class 2 will operate at the company level, class 3 will be attached to the battalion and Class 4 to the brigade commander. Each FCS brigade would have 36 class-1, 36 class-2, 12 class-3 and 16 class-4 aircraft. The FCS program generally has been described as a network of ground and air vehicles—both manned and un-piloted. The most "undefined" of the four classes of UAVs is the brigade-level aircraft. The funding and design of the FCS class-4 UAVs closely are tied to the Army's next-generation helicopter, the Comanche. The service cut more than 600 helicopters out of the program (about half of the total), on the assumption that they would be replaced with UAVs.

C. UNMANNED GROUND VEHICLES (UGV)

Representative Picture	Description
	**Armed Robotic Vehicle** (ARV) - Assault - RSTA The ARV is a 5 to 6 ton unmanned ground vehicle (UGV) that performs an armed Reconnaissance, Surveillance, and Target Acquisition (RSTA) mission. The ARV will be part of an organization of vehicles, sensors, C2 hardware and software systems, and communications systems. The ARV assault incorporates a turret system capable of launching missiles such as the Common Missile or

	Hellfire and operating a medium caliber gun system such as the 30mm Mk 44 Chain Gun. The ARV provides mobility sufficient to maneuver with the FCS force, and must be compatible with C-130 and CH-47 (internal) deployment. The ARV provides semi-autonomous navigation and mission equipment operations, with man-in-the-loop weapon fire authorization via the C4ISR network.
	**Multifunction Utility/Logistics Equipment Vehicle** (MULE) The MULE is an unmanned platform that provides transport of equipment and/or supplies in support of dismounted maneuver. There are three variants of the MULE. These are MULES designed for 1) transport, 2) Air assault, and 3) Countermine use. Anything else that's mission-essential but not built in to the individual soldier system will be carried on a "robotic mule." The mule will assist with not only taking some of the load carriage off the individual soldier, but he also provides a host of other functions. Primarily water generation (and) water purification. It's a recharging battery station for all the individual Objective Force Warriors in the squad. It acts as a weapons platform. It has day and night thermal, infrared and forward-looking imaging systems inside the nose of the mule, as well as chemical-biological sensors. The mule can also communicate with unmanned aerial vehicles to give the squad members a true 360-degree image of the battlefield
	**Small (Manpackable) UGV** The Soldier UGV (SUGV) is a man-packable small robot system, weighing less than 30 lbs, used for Urban Operations environments and subterranean features to remotely investigate the threat obstacles, structures and the structural integrity of facilities and utilities. SUGV systems will be highly mobile for dismounted forces and will be capable of being re-configured for other missions by adding or removing sensors, modules, mission payloads, and/or subsystems.

D. INTELLIGENT MUNITIONS

Representative Picture	Description
	NetFires is a technology demonstration program focused on beyond line-of-sight fires for the Army's Future Combat Systems. The program is DARPA managed using combined DARPA-Army S&T funding. Proof of principle test flights are scheduled to begin in FY03. The programs technology demonstration elements include: container launch unit (CLU); loitering attack missile (LAM); and precision attack missile (PAM). The Netfires (formerly Advanced Fire Support System (AFSS)) program will develop and test a containerized, platform-independent multi-mission weapon concept as an enabling technology element for FCS. NetFires will provide rapid response and lethality in packages requiring significantly fewer personnel, decreased logistical support and lower life-cycle costs, while increasing survivability compared to current direct fire gun and missile artillery. The original concept was called "Rockets in a Box."

APPENDIX B

A. FUTURE COMBAT SYSTEMS DEVELOPMENT AND DEMONSTRATION SCHEDULE PLANS

This thesis examines three plans for the FCS System Development and Demonstration schedule to contrast the relative schedule risks. The three plans, developed by the General Accounting Office (GAO) in its review of the FCS program (GAO, 2003) are:

- **Baseline Plan.** Proceed with the concurrent development plan developed by the FCS Project Executive Office – Ground Combat Systems (PEO-GCS).
- **Alternate Plan 1:** Mature critical technologies first to mitigate risk, and then proceed with the concurrent development plan developed by the FCS PEO. System integration and test tasks are assumed to proceed with lower schedule risk under Option 1 than under the baseline plan.
- **Alternate Plan 2:** Develop the C4ISR infrastructure before initiating the development of other FCS systems. System integration and test tasks are assumed to proceed with lower schedule risk under alternate plan 2 than under the baseline plan.

Schedules for each of these tasks were developed in Microsoft Project 2000. Summary reports of the key tasks are shown below.

Summary description tasks are bold text. Zero duration tasks are milestones.

1. Baseline Plan: Proceed with Current Plan

Project Start Date: 1/01/03
Project Finish Date: 30/10/12

ID	Task_Name	Estimated Cost ($M)	Duration (weeks)	Successors
1	Notional Start	0.00	0	24,13,3
2	**Major Events**			
3	Milestone B complete	0.00	0	4,67,37,29,25,14
4	SFR (System Functional Review)	0.00	0	5,16,26
5	SoS PDR complete	0.00	0	6,17
6	SoS CDR complete	0.00	0	7
7	Facilitation	0.00	0	8,95
8	LL IPR Waiver	0.00	0	9,97
9	IPD (Milestone C)	0.00	0	10,77
10	IOC	0.00	0	11,32
11	UA	0.00	0	101
12	**SoS Definition and Design**			
13	Systems Engineering	571.42	104	5
14	Systems Design	1428.57	260	10
15	**Prototype Systems Build and Test**			
16	1st Variant PDC (Preliminary Design Complete)	0.00	0	17
17	Last Variant PDC (Preliminary Design Complete)	0.00	0	18,20,44
18	Long Lead Prototype	800.00	52	19,21
19	Prototype Integration and Assembly	1200.00	78	22
20	First Variant CDC (Critical Design Complete)	0.00	0	69,21
21	Last Variant CDC (Critical Design Complete)	0.00	0	22,6
22	Final Prototype	0.00	0	97,8
23	**C4ISR Software and Platform**			
24	SW Build 1	507.93	104	27,44
25	SW Build 2	634.92	130	27,34,69,31,46,52,59
26	SW Build 3	825.39	169	28,52,59
27	SW Build 4	571.42	117	9,63,59
28	SW Build 5	507.93	104	83,89,64
29	SIL Delivery 1 (System Integration Lab)	253.96	52	68,33,30
30	SIL Delivery 2	253.96	52	69,31,27,52
31	SIL Delivery 3	253.96	52	32,28
32	SW Update	190.47	39	11,80
33	Software PDR complete	0.00	0	34,5
34	Software CDR complete	0.00	0	6
35	**Integrated Test Program**			
36	**IPS1 (Integration Phase SDD 1)**			
37	SoSIL Development	280.99	51	38,39,30
38	Integration	71.62	13	41,5,40
39	Sims Delivered	0.00	0	40
40	IT/UT	71.62	13	42
41	TRR	0.00	0	42

42	Analysis	71.62	13	45,44
43	**IPS2**			
44	Integration	280.99	51	47,6,46
45	Early Emulators Delivered	0.00	0	46
46	IT/UT	71.62	13	48
47	TRR	0.00	0	48
48	Analysis	71.62	13	50,51,28
49	**IPS3**			
50	Integration	209.36	38	53,52
51	Initial DP Prime Items delivered	0.00	0	52
52	IT /UT	71.62	13	54,55
53	TRR	0.00	0	54,55
54	Analysis	104.68	19	58,8
55	User Trial	11.01	2	57
56	**IPS4**			
57	Integration	187.32	34	60,59
58	Initial System Deliveries	0.00	0	59
59	IT/UT	71.62	13	61,63,72
60	TRR	0.00	0	61
61	Analysis	71.62	13	9
62	**IPS5**			
63	Integration	209.37	38	64
64	IMT	71.63	13	65
65	Analysis	71.63	13	77,100
66	**SoS Testing and Integration**			
67	Phase 1 : Integration & Test SDD (Simulation)	183.75	78	70,5
68	Phase 2 : HW/SW	214.37	91	6,95
69	Phase 3 : Prototype	214.37	91	72,57,8
70	Integration / Qualification / Live Fire Tests	489.99	208	73,9,76
71	**Test Events and Milestones**			
72	LUT 1	4.71	2	73
73	LUT 2	4.71	2	77,79,74,98,99
74	IOT (Initial Operational Test) Phase 1	47.11	20	10,75
75	IOT Phase 2	44.76	19	80
76	Integration and Test Production	214.37	91	10,80
77	FUSL	244.99	104	80,11
78	Training and Fielding	244.99	104	80
79	IOTE 1	61.25	26	80
80	IOTE 2	30.62	13	11
81	**Combat Systems Testing**			
82	**Phase 1: LRIP Prime Items**			
83	Integration	634.15	39	85,89,100
84	LRIP PI for SoSIL	0.00	0	85
85	LRIP PI for TFT Delivered	0.00	0	86
86	Testing	211.38	13	87,90
87	Analysis	211.38	13	92,74,79
88	**Phase 2: LRIP Late LRIP PI**			
89	Integration	520.33	32	91
90	LRIP PI for SoSIL	0.00	0	91
91	LRIP PI for TFT Delivered	0.00	0	92
92	Testing	211.38	13	93
93	Analysis	211.38	13	11,10,32

94	**Production**			
95	Facilitation (Pre-LL Production)	682.93	52	100,84,96
96	Facilitation (LL Production)	1195.12	91	100,84
97	Long Lead Lot 1	682.93	52	98,99,100,9,83,84,76
98	Lot 1	1024.39	78	79,78
99	Lot 2	1707.32	130	11,80
100	Lot 3	1707.32	130	11,80
101	Notional End Task	0	0	

2. Alternate Plan 1: Mitigate High Risk Technologies First

Project Start Date: 1/01/03
Project Finish Date: 22/01/13

ID	Task_Name	Estimated Cost ($M)	Duration (Weeks)	Successors
1	Notional Start	0.00	0	57,13,3
2	**Major Events**			
3	Milestone B complete	0.00	0	4,100,70,62,58,14
4	SFR (System Functional Review)	0.00	0	5,49,59
5	SoS PDR complete	0.00	0	6,50
6	SoS CDR complete	0.00	0	7
7	Facilitation	0.00	0	8,128
8	LL IPR Waiver	0.00	0	9,130
9	IPD (Milestone C)	0.00	0	10,110
10	IOC	0.00	0	11,65
11	UA	0.00	0	134
12	**SoS Definition and Design**			
13	Systems Engineering	571.43	104	5
14	Systems Design	1428.57	260	10
15	**Prototype Systems Build and Test**			
16	**TRL Mitigation (Technology Readiness Level)**			
17	**KPP 1: Joint Interoperability**			
18	Interface & Information Exchange	113.24	65	4
19	**KPP 2: Networked Battle Command**			
20	Security Systems & Algorithms	249.13	143	6
21	Quality of Service Algorithms	67.94	39	3
22	Wideband Waveforms	181.18	104	5
23	Multispectral Sensors & Seekers	90.59	52	3
24	Combat Identification	22.65	13	3
25	Sensor/Data Fusion & Data Compression	67.94	39	3
26	**KPP 3: Networked Lethality**			
27	Dynamic Sensor-Shooter Pairing / Fire Control	90.59	52	3
28	LOS/BLOS/NLOS Precision Munitions Guidance	271.78	156	6
29	Aided Target Recognition	67.94	39	3
30	Auto Target Recognition	181.18	104	5
31	Recoil Management & Lightweight Components	90.59	52	3
32	Distributed Collaboration of Manned / Unmanned Vehicles	226.48	130	5
33	Rapid Battle Damage Assessment	67.94	39	3
34	**KPP 4: Transportability**			
35	High Power Density / Fuel Efficient Propulsion	90.59	52	3
36	**KPP 5: Sustainability / Reliability**			
37	Embedded Predictive Logistic Sensors / Algorithms	90.59	52	3
38	Water Generation and Purification	90.59	52	3

39	**KPP 6: Training**			
40	Computer Generated Forces	22.65	13	3
41	Tactical Engagement Simulation	45.30	26	3
42	**KPP 7: Survivability**			
43	Active Protection System	22.65	13	3
44	Signature Management	90.59	52	3
45	Lightweight hull and Vehicle Armour	10.45	6	3
46	Power Distribution and Control	10.45	6	3
47	Advanced Countermine Technology	226.48	130	5
48	High Density Packaged Power	10.45	6	3
49	1st Variant PDC (Preliminary Design Complete)	0.00	0	50
50	Last Variant PDC (Preliminary Design Complete)	0.00	0	51,53,77
51	Long Lead Prototype	600.00	52	52,54
52	Prototype Integration and Assembly	900.00	78	55
53	First Variant CDC (Critical Design Complete)	0.00	0	102,54
54	Last Variant CDC (Critical Design Complete)	0.00	0	55,6
55	Final Prototype	0.00	0	130,8
56	**C4ISR Software and Platform**			
57	SW Build 1	380.95	104	60,77
58	SW Build 2	476.19	130	60,67,102,64
59	SW Build 3	619.05	169	61
60	SW Build 4	428.57	117	9,96
61	SW Build 5	380.95	104	116,122
62	SIL Delivery 1 (System Integration Lab)	190.48	52	101,66,63
63	SIL Delivery 2	190.48	52	102,64,60
64	SIL Delivery 3	190.48	52	65,61
65	SW Update	142.86	39	11
66	Software PDR complete	0.00	0	67,5
67	Software CDR complete	0.00	0	6
68	**Integrated Test Program**		330	
69	**IPS1 (Integration Phase SDD 1)**		90	
70	SoSIL Development	219.20	51	71,72,63
71	Integration	74.50	13	74,5,73
72	Sims Delivered	0.00	0	73
73	IT/UT	74.50	13	75
74	TRR	0.00	0	75
75	Analysis	74.50	13	78,77
76	**IPS2**			
77	Integration	263.61	46	80,6,79
78	Early Emulators Delivered	0.00	0	79
79	IT/UT	74.50	13	81
80	TRR	0.00	0	81
81	Analysis	74.50	13	83,84,61
82	**IPS3**			
83	Integration	200.57	35	86,85
84	Initial DP Prime Items delivered	0.00	0	85
85	IT/UT	74.50	13	87,88

86	TRR	0.00	0	87,88
87	Analysis	108.88	19	91
88	User Trial	11.46	2	90,8
89	**IPS4**			
90	Integration	177.65	31	93,92
91	Initial System Deliveries	0.00	0	92
92	IT/UT	74.50	13	94,96,105
93	TRR	0.00	0	94
94	Analysis	74.50	13	9
95	**IPS5**		61	
96	Integration	200.57	35	97
97	IMT	74.50	13	98
98	Analysis	74.50	13	110,133
99	**SoS Testing and Integration**			
100	Phase 1 : Integration & Test SDD (Simulation)	183.75	78	103,5
101	Phase 2 : HW/SW	214.37	91	6,128
102	Phase 3 : Prototype	214.37	91	105,90,8
103	Integration / Qualification / Live Fire Tests	489.99	208	106,9,109
104	**Test Events and Milestones**			
105	LUT 1	4.71	2	106
106	LUT 2	4.71	2	110,112,107
107	IOT (Initial Operational Test) Phase 1	47.11	20	10,108
108	IOT Phase 2	44.76	19	113
109	Integration and Test Production	214.37	91	10,113
110	FUSL	244.99	104	113,11
111	Training and Fielding	244.99	104	113
112	IOTE 1	61.25	26	113
113	IOTE 2	30.62	13	11
114	**Combat Systems Testing**			
115	**Phase 1: LRIP Prime Items**			
116	Integration	634.15	39	118,122
117	LRIP PI for SoSIL	0.00	0	118
118	LRIP PI for TFT Delivered	0.00	0	119
119	Testing	211.38	13	120,123
120	Analysis	211.38	13	125,107,112
121	**Phase 2: LRIP Late LRIP PI**			
122	Integration	520.33	32	124
123	LRIP PI for SoSIL	0.00	0	124
124	LRIP PI for TFT Delivered	0.00	0	125
125	Testing	211.38	13	126
126	Analysis	211.38	13	11,10
127	**Production**			
128	Facilitation (pre-LL production)	833.33	65	129
129	Facilitation (LL Production)	1166.67	91	133,117
130	Long Lead Lot 1	666.67	52	131,132,133,9,116,117
131	Lot 1	1000.00	78	112,111
132	Lot 2	1666.67	130	11,113
133	Lot 3	1666.67	130	11,113
134	Notional End	0.00	0	

3. Alternate Plan 2: Develop C4ISR Infrastructure First

Project Start Date: 1/01/03
Project Finish Date: 15/04/14

ID	Task_Name	Estimated Cost ($M)	Duration (weeks)	Successors
1	Notional Start	0	0	24,13,3
2	**Major Events**			
3	Milestone B complete	0	0	4,67,37,29,25,14
4	SFR (System Functional Review)	0	0	5,26
5	SoS PDR complete	0	0	6,16
6	SoS CDR complete	0	0	7,17
7	Facilitation	0	0	8,95
8	LL IPR Waiver	0	0	9,97,21
9	IPD (Milestone C)	0	0	10,77
10	IOC	0	0	11,32
11	UA	0	0	101
12	**SoS Definition and Design**			
13	Systems Engineering	571.43	104	5
14	Systems Design	1428.57	260	10
15	**Prototype Systems Build and Test**			
16	1st Variant PDC (Preliminary Design Complete)	0	0	17
17	Last Variant PDC (Preliminary Design Complete)	0	0	18
18	Long Lead Prototype	800	52	19,20,21
19	Prototype Integration and Assembly	1200	78	22
20	First Variant CDC (Critical Design Complete)	0	0	95
21	Last Variant CDC (Critical Design Complete)	0	0	22
22	Final Prototype	0	0	57,69,97,96
23	**C4ISR Software and Platform**			
24	SW Build 1	507.94	104	27,44
25	SW Build 2	634.92	130	27,34,69,31,46,52,59
26	SW Build 3	825.4	169	28,52,59
27	SW Build 4	571.43	117	9,63,59
28	SW Build 5	507.94	104	83,89,64
29	SIL Delivery 1 (System Integration Lab)	253.97	52	68,33,30
30	SIL Delivery 2	253.97	52	69,31,27,52
31	SIL Delivery 3	253.97	52	32,28
32	SW Update	190.48	39	11,80
33	Software PDR complete	0	0	34,5
34	Software CDR complete	0	0	6
35	**Integrated Test Program**			
36	**IPS1 (Integration Phase SDD 1)**			
37	SoSIL Development	280.99	51	38,39,30
38	Integration	71.63	13	41,5,40
39	Sims Delivered	0	0	40
40	IT/UT	71.63	13	42
41	TRR	0	0	42

42	Analysis	71.63	13	45,44
43	**IPS2**			
44	Integration	280.99	51	47,6,46
45	Early Emulators Delivered	0	0	46
46	IT/UT	71.63	13	48
47	TRR	0	0	48
48	Analysis	71.63	13	50,51,28
49	**IPS3**			
50	Integration	209.37	38	53,52
51	Initial DP Prime Items delivered	0	0	52
52	IT/UT	71.63	13	54,55
53	TRR	0	0	54,55
54	Analysis	104.68	19	58,8
55	User Trial	11.02	2	57
56	**IPS4**			
57	Integration	187.33	34	60,59
58	Initial System Deliveries	0	0	59
59	IT/UT	71.63	13	61,63,72
60	TRR	0	0	61
61	Analysis	71.63	13	9
62	**IPS5**			
63	Integration	209.37	38	64
64	IMT	71.63	13	65
65	Analysis	71.63	13	77,100
66	**SoS Testing and Integration**			
67	Phase 1 : Integration & Test SDD (Simulation)	183.75	78	70,5
68	Phase 2 : HW/SW	214.37	91	6,95,57
69	Phase 3 : Prototype	214.37	91	72
70	Integration / Qualification / Live Fire Tests	489.99	208	73,9,76
71	**Test Events and Milestones**			
72	LUT 1	4.71	2	73
73	LUT 2	4.71	2	77,79,74,98,99,76
74	IOT (Initial Operational Test) Phase 1	47.11	20	10,75
75	IOT Phase 2	44.76	19	80
76	Integration and Test Production	214.37	91	10,80
77	FUSL	244.99	104	80,11
78	Training and Fielding	244.99	104	80
79	IOTE 1	61.25	26	80
80	IOTE 2	30.62	13	11
81	**Combat Systems Testing**			
82	**Phase 1: LRIP Prime Items**			
83	Integration	634.15	39	85,89,100
84	LRIP PI for SoSIL	0	0	85
85	LRIP PI for TFT Delivered	0	0	86
86	Testing	211.38	13	87,90
87	Analysis	211.38	13	92,74,79
88	**Phase 2: LRIP Late LRIP PI**			
89	Integration	520.33	32	91
90	LRIP PI for SoSIL	0	0	91

91	LRIP PI for TFT Delivered	0	0	92
92	Testing	211.38	13	93
93	Analysis	211.38	13	11,10,32
94	**Production**			
95	Facilitation (Pre-LL Production)	682.93	52	100,84,96
96	Facilitation (LL Production)	1195.12	91	100,84
97	Long Lead Lot 1	682.93	52	98,99,100,9,83,84,76
98	Lot 1	1024.39	78	79,78
99	Lot 2	1707.32	130	11,80
100	Lot 3	1707.32	130	11,80
101	Notional end	0	0	

B. ANNUAL FCS BUDGET BANDS BY COMPLETION YEAR

	Year of Project Completion									
Year	**2010** Min–Max	**2011** Min–Max	**2012** Min–Max	**2013** Min–Max	**2014** Min–Max	**2015** Min–Max	**2016** Min–Max	**2017** Min–Max	**2018** Min–Max	**2019** Min–Max
2003	221–1,161	175–919	142–746	118–621	101–529	87–458	77–403	69–361	62–327	57–300
2004	595–3,125	482–2,530	398–2,087	335–1,760	288–1,511	251–1,318	222–1,168	200–1,049	182–954	167–878
2005	798–4,192	676–3,551	576–3,026	498–2,614	435–2,285	385–2,023	345–1,812	313–1,643	287–1,505	266–1,394
2006	807–4,237	732–3,841	655–3,437	586–3,078	527–2,766	477–2,502	434–2,280	399–2,095	370–1,941	346–1,815
2007	672–3,528	667–3,502	637–3,343	598–3,142	558–2,929	519–2,726	484–2,541	453–2,378	426–2,238	403–2,118
2008	477–2,506	530–2,785	549–2,884	548–2,878	535–2,809	516–2,710	495–2,599	474–2,488	454–2,385	437–2,293
2009	294–1,544	374–1,965	427–2,243	458–2,406	473–2,482	476–2,499	472–2,479	465–2,440	456–2,393	446–2,344
2010	159–833	237–1,242	303–1,589	353–1,854	388–2,039	411–2,158	424–2,229	431–2,265	434–2,280	435–2,283
2011		135–708	196–1,031	252–1,325	299–1,568	335–1,757	362–1,899	381–2,002	396–2,077	406–2,131
2012			117–615	168–881	216–1,132	258–1,353	293–1,539	322–1,691	346–1,815	365–1,916
2013				104–547	147–771	188–989	227–1,191	261–1,370	291–1,526	317–1,662
2014					94–495	131–687	168–881	203–1,066	236–1,237	266–1,394
2015						87–455	119–624	152–798	185–969	216–1,133
2016							81–424	110–575	140–733	170–894
2017								76–400	102–537	130–684
2018									73–381	97–508
2019										70–367
Total	4,024–21,126	4,008–21,042	4,000–21,000	4,020–21,105	4,060–21,316	4,121–21,636	4,204–22,069	4,309–22,620	4,438–23,299	4,593–24,114

Annual FCS Budget Bands by Completion Year (2004 $ Millions)

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APPENDIX C

A. JAVA CODE FOR UNCONSTRAINED REACHING ALGORITHM

```
/**
 * Unconstrained Reaching Algorithm
**/

import java.util.*;
import java.io.*;

public class reach2 {
   // Class Constants
   public static final double mode_a_low  = 1.15;
   public static final double mode_a_med  = 1.20;
   public static final double mode_a_high = 1.25;
   public static final double pr_tML_low  = 0.60;
   public static final double pr_tML_med  = 0.70;
   public static final double pr_tML_high = 0.80;
   public static final int monteCarloIterations = 60000;
   private static PrintWriter outputStream = null;


   public static void main(String args[]) {
      // Text Input Tools
      String          inputString;
      BufferedReader  inputUnit;
      StringTokenizer tk;
      long startTime, finishTime, elapsedTime;

      try{
         outputStream = new PrintWriter(new FileOutputStream ("num.txt"));
      }
      catch (FileNotFoundException e){
         System.out.println("Error opening output file");
         System.exit(0);
      }

      //Network data structure elements: forward-star
      int i, j, t, k, n, m, T, r;
      int [] point;
      int [] tail;
      int [] head;
      int [] duration;
      int [] origDuration;

      // random duration variables
      int [] risk;
      double [] min;
      double [] shape;
      double [] scale;
      double [] mode;
      double [] prob;

      //Check usage (at least one command line argument)
      if(args.length==0) {
         System.out.println("\nUsage: java reach2 <filename>");
         return;
      }

      k=0;
      try {
         inputUnit = new BufferedReader(new FileReader(args[0]));
         if((inputString = inputUnit.readLine())==null){
            System.out.println("Premature end of file encountered. k="+k);
         return;
```

```
    }

    tk = new StringTokenizer(inputString);
    n = Integer.parseInt(tk.nextToken());
    m = Integer.parseInt(tk.nextToken());

    point        = new int [n+2];
    tail         = new int [m+1];
    head         = new int [m+1];
    origDuration = new int [m+1];
    duration     = new int [m+1];
    risk         = new int [m+1];
    min          = new double [m+1] ;
    shape        = new double [m+1] ;
    scale        = new double [m+1] ;
    prob         = new double [m+1] ;
    mode         = new double [m+1] ;

    point[0]     =1;
    T            =0;

    for(k=1; k<=m; k++){
       if((inputString = inputUnit.readLine())==null){
          System.out.println("Premature end of file encountered. k="+k);
          return;
       }

       tk = new StringTokenizer(inputString);
       i = Integer.parseInt(tk.nextToken());
       j = Integer.parseInt(tk.nextToken());
       t = Integer.parseInt(tk.nextToken());
       r = Integer.parseInt(tk.nextToken()); // 1 = Low Risk,
                                             // 2 = Med Risk,
                                             // 3 = High Risk

       if(t > T){
          T=t;
       }

       point[i]++;
       tail[k]         =i;
       head[k]         =j;
       origDuration[k] =t;
       risk[k]         =r;

       // Assign standard risk based values from CAIG to each activity.
       if(r == 1){
          // Low Risk
          mode[k] = mode_a_low;
          prob[k] = pr_tML_low;
       }else if (r == 2){
          // med risk
          mode[k] = mode_a_med;
          prob[k] = pr_tML_med;
       }else if (r == 3){
          // high risk
          mode[k] = mode_a_high;
          prob[k] = pr_tML_high;
       }else{
          System.out.println("Invalid risk code at k = "+k);
          return;
       }
 } // close data input loop loop
 inputUnit.close();
 } catch(FileNotFoundException e) {
    System.out.println(e);
    return;
 } catch(IOException e){
    System.out.println(e);
    return;
 } catch(NoSuchElementException e) {
```

```
   System.out.println("No data found: k="+k);
   return;
}

//Clean up point array
for(i=1; i <= n+1; i++) {
   point[i]=point[i-1]+point[i];
}
for(k=1; k <= m; k++) {
   i=tail[k];
   point[i]--;
}
System.out.println(" Finished reading data file...");

//***************  END DATA ENTRY - START REACH ALGORITHM

startTime = System.currentTimeMillis();

System.out.println(" Opening Simulation Loop - writing to file");
int monteCarloCounter;
monteCarloCounter = 1;

// Open the Monte Carlo Loop
while ( monteCarloCounter < monteCarloIterations){

  /* Change the Project Duration to reflect the project activity risk.
   * This will give one realization of the activity duration, which is
   * based on the Weibull timeribution.  Assumptions are those used by
   * the Cost Analysis Improvement Group (CAIG), PA&E.
   */
   for(k = 1; k <= m; k++){
      min[k] = origDuration[k] / mode[k];
      if (origDuration[k] > 0) {
         shape[k]    = 1 / (Math.log(prob[k])+1);
         scale[k]    = (origDuration[k]-min[k]) /
                          Math.pow((1-1/shape[k]),(1/shape[k]));
             duration[k] = (int)Math.ceil( (min[k] + scale[k]*
                          (Math.pow((-Math.log(1-Math.random())), (1/shape[k])))));
      }  else {
         shape[k] = 0;
         scale[k] = 0;
      }
   }

   // Data structure elements
   int     s;
   int     cardTEMP, cardPERM, minTime;
   boolean [] TEMP;
   boolean [] PERM;
   int     [] pred;
   int     [] activityTime;

   // Algorithm code starts here
   pred         = new int[n + 1];
   activityTime = new int[n + 1];
   TEMP         = new boolean[n + 1];
   PERM         = new boolean[n + 1];

   s        = 1;
   cardTEMP = 0;
   cardPERM = 0;

   // initialize the temp and perm cardinality flags.
   while (cardTEMP < n) {
      cardTEMP ++;
      activityTime[cardTEMP] = 0;
      TEMP[cardTEMP] = true;
      PERM[cardTEMP] = false;
   }

   activityTime[s] = 0;
```

```
        pred[s]         = 0;

        // Calculate the longest path.
        for (i=0; i < n; i++) {
           minTime   = 0;
           PERM[i]   = true;
           cardPERM ++;
           TEMP[i]   = false;
           cardTEMP --;
           k         = point[i];

           while (k < point[i + 1]) {
              j = head[k];
              if (activityTime[j] < (activityTime[i] + duration[k])) {
                 activityTime[j]  = (activityTime[i] + duration[k]);
                 pred[j] = i;
              }
              k++;
           }
        }        // End reaching code

        int maxsp;
        int maxnode;

        /*Now determine the results*/
        maxsp=0;
        maxnode=0;
        for(i=1; i<=n; i++){
           if(activityTime[i] > maxsp && activityTime[i] > 0){
              maxsp = activityTime[i];
              maxnode = i;
           }
        }
        i = maxnode;
        j = 0;

        outputStream.println(maxsp);

        //System.out.println(" Hops in longest SP: "+j);
        //System.out.println(" activity duration: "+monteCarloCounter+" is: "+
                                      maxsp);
        if (monteCarloCounter == monteCarloIterations/2){
           System.out.println("    ...Half way through the simulation...");
        }
        monteCarloCounter++;
     }  // close of Monte Carlo Loop

     System.out.println(" finished the Simulation ");
     outputStream.close();

     finishTime  = System.currentTimeMillis();
     elapsedTime = finishTime-startTime;
     System.out.println(" Elapsed time = "+ elapsedTime);
  }
}
```

APPENDIX D

A. COMPLETE ILP FORMULATION

1. Index Use

$y \in Y$	fiscal year (alias *yh, yf*) (20)
$i \in I$	task (alias j) (~100)
$\ell \in I$	distinguished, last task in project
$m \in M$	planning month (~240)
$m \in M(y)$	month in fiscal year *y*
$s_i \in S_i \subseteq M$	start month for task *i*
$d_i \in D_i$	task *i* duration in months
$1 \le p_i \le d_i$	period of ongoing task *i*

2. Data

$\underline{budget}_{y,yf}$	Lower cost range during fiscal year *y* if program finished in fiscal year *yf* [cost]
$\overline{budget}_{y,yf}$	Upper cost ranges during fiscal year *y* if program finished in fiscal year *yf* [cost]
$cost_{id_ip_i}$	Cost of ongoing task *i* with duration *d* during elapsed month *p* [cost]
pen_under	Cost per unit of negative cumulative budget range violation [months/cost]
pen_over	Cost per unit of positive cumulative budget range violation [months/cost]

3. Variables

$X_{is_id_i}$ Binary variable, which is set to 1 if task i is started in month s with duration d and set to 0 otherwise

Q_{yf} Binary variable, which is set to 1 if finish year of program is year yf, and set to 0 otherwise.

$UNDER_y$ When expenditures through fiscal year y are compared with desired lower ranges on total budgets, this variable measures lower-range violations.

$SLACK_y$ When expenditures through fiscal year y is compared with desired lower and upper ranges on total budgets, this variable measures unspent funds below upper-range violation.

$OVER_y$ When expenditures through fiscal year y are compared with desired upper ranges on total budgets, this variable measures upper-range violations.

4. Formulation

$$\underset{\substack{X,Q\\UNDER\\SLACK\\OVER}}{MIN} \quad \sum_{\substack{s_\ell \in S_\ell, d_\ell \in D_\ell\\ \wedge s_\ell + d_\ell - 1 \le \|M\|}} (s_\ell + d_\ell - 1) X_{\ell s_\ell d_\ell}$$

$$+ \sum_{y \in Y} \left(pen\_under\, UNDER_y + pen\_over\, OVER_y \right) \qquad (F1)$$

$$s.t. \quad \sum_{\substack{s_i \in S_i\\ d_i \in D_i \wedge s_i + d_i - 1 \le \|M\|}} X_{is_id_i} = 1 \qquad \forall i \in I \qquad (F2)$$

$$X_{\ell s_\ell d_\ell} \le Q_{yf} \qquad \forall yf \in Y, \; s_\ell \in S_\ell, d_\ell \in D_\ell \wedge s_\ell + d_\ell - 1 \in M(yf) \qquad (F3)$$

$$\sum_{yf \in Y} Q_{yf} = 1 \qquad (F4)$$

$$\sum_{\substack{yh \le y, m \in M(yh)\\ i \in I, s_i \in S_i, d_i \in D_i\\ \wedge m - s_i + 1 \ge 1\\ \wedge m - s_i + 1 \le d_i\\ \wedge s_i + d_i - 1 \le \|M\|}} cost_{id_i(m - s_i + 1)} X_{is_id_i} + UNDER_y + SLACK_y - OVER_y$$

$$= \sum_{\substack{yh \le y,\\ yf \in Y \wedge yf \ge y}} \left(\overline{budget}_{yh,yf} \right) Q_{yf} \qquad \forall y \in Y \qquad (F5)$$

$$SLACK_y \le \sum_{\substack{yh \le y,\\ yf \in Y \wedge yf \ge y}} \left(\overline{budget}_{yh,yf} - \underline{budget}_{yh,yf} \right) Q_{yf} \qquad \forall y \in Y \qquad (F6)$$

$$\sum_{s_i \in S_i, d_i \in D_i \wedge s_i + d_i - 1 < s_j} X_{is_id_i} \ge X_{js_jd_j} \qquad \forall (i,j) \in A, \forall s_j \in S_j, d_j \in D_j \wedge s_j + d_j - 1 \le \|M\| \wedge s_j > \underset{s_i \in S_i, d_i \in D_i}{MIN} (s_i + d_i - 1) \qquad (F7)$$

$$X_{is_id_i} \in \{0,1\} \qquad \forall i \in I, s_i \in S_i, d_i \in D_i \qquad (F8)$$

$$Q_{yf} \in \{0,1\} \qquad \forall yf \in Y \qquad (F9)$$

$$UNDER_y \ge 0; \; SLACK_y \ge 0; \; OVER_y \ge 0 \qquad \forall y \in Y \qquad (F10)$$

5. Verbal Description

The objective function (F1) expresses total planned project duration in months, plus an elastic violation term for any violation of budget ranges over the planning horizon.

Constraints:

(F2) Each partition constraint requires that exactly one start month and duration be selected for each task.

(F3) Each constraint permits the last project task to be completed in a fiscal year only if that fiscal year has been selected for project completion.

(F4) A partition constraint requires that exactly one project completion year be selected.

(F5) Each constraint accumulates expenditures from the first fiscal year through a current fiscal year and determines whether the cumulative budget ranges have been satisfied, or violated.

(F6) Each constraint limits cumulative slack budget by the program budget determined by finish year.

(F7) Each constraint ensures that for a pair of tasks sharing a partial order precedence, the predecessor task must be completed before the successor task can start.

(F8) Selections to be binary.

(F9) Selections to be binary.

(F10) Limits budget range violations.

6. Model Discussion

Partition constraints ($F2$) are generalized upper bounds (GUBs) (Dantzig and Van Slyke, 1967). Further, each of these GUB partitions exhibit contiguous ones by row, a desirable property (Ayik, A, 2000). The budget constraint ($F3$) would better be stated in canonical form more amenable to a linear programming solver (Brown, Dell and Wood, 1997), e.g.:

$$\sum_{\substack{yh \le y, m \in M_y \\ i \in I, s_i \in S_i, d_i \in D_i \\ \wedge s_i \ge m+1-d_i \\ \wedge s_i \le \|M\|+1-d_i}} cost_{id_i(m-s+1)} X_{is_id_i} + UNDER_y + SLACK_y - OVER_y$$

$$= \sum_{yh \le y} \overline{budget}_{yh} \qquad \forall y \in Y \qquad (F3^*)$$

Unfortunately, an algebraic modeling language (in our case, GAMS) requires three orders of magnitude more time to generate this constraint than an integer linear programming solver (in our case, OSL) needs to solve it. Accordingly, the mathematically equivalent, easier-to-generate, but harder-to-solve alternate form ($F3^*$) has been used.

7. Appendix D References

Ayik, M. (2000). *Exploiting Consecutive Ones Structure in the Set Partitioning Problem*, Ph.D. Dissertation in Operations Research, Naval Postgraduate School, Monterey, CA, December.

Brown, G.G., Dell, R.F., and Wood, R.K. (1997). "Optimization and Persistence", *INTERFACES*, **27**, pp. 15-37.

Dantzig, G., and Van Slyke, R. (1967). "Generalized Upper Bounding Techniques", *Journal of Computing and System Science*, **1**, pp. 213-226.

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APPENDIX E

A. GAMS IMPLEMENTATION

This appendix contains the code for the GAMS implementation of the FCS Scheduler ILP model.

```
$INLINECOM { }
OPTIONS
  SOLPRINT =    OFF,
  DECIMALS =      1,
  LIMCOL   =     10,
  LIMROW   =    100,
  RESLIM   =   1000, {max seconds}
  ITERLIM  =  99999, {max pivots}
  OPTCR    =  0.05 , {relative integrality tolerance}
  LP       =  cplex,
  RMIP     =  cplex,
  MIP      =  cplex; {OSL, CPLEX, XA, ... }

 SETS
   i          "task"
  /
$include fcs_tasks.txt
$ontext
$include toy_tasks.txt
$offtext
  last_task   {ultimate successor}
  /
   r          "activity risk levels"
  /
   r1*r3
  /

   y          "fiscal year"
  /
   fy2003*fy2019    { moderation is a virtue, do not over-extend planning horizon }
  /
   fm         "fiscal month"
  /
   oct
   nov
   dec
   jan
   feb
   mar
   apr
   may
   jun
   jul
   aug
   sep
  /

  static_arcs(i,i)   "pairwise precedence relations"
  /
$include fcs_arcs.txt
$ontext
$include toy_arcs.txt
$offtext
  /
  arcs(i,i)    "dynamic (static)  set with pairwise precedence relations and last_task"

   d          "duration months"
  /
   m000*m204
```

```
 /

 m(d)        "planning months"
 /
  m001*m204
 /
 ;

alias(i,j) ;
   {  i            task, or predecessor task                  }
   {    j          successor task                             }

alias(m,s,si,sj,mw,p) ;
   {  m              fiscal calendar month, =1,2,...ub          }
   {    s            task start month                           }
   {      si         task i start month                         }
   {         xj      task j start month                         }
   {            mw   moving window start                        }
   {               p number of elapsed months since task start  }

alias(d,di,dj)
   {  d              task duration (starts with zero months     }
   {    di           task i duration                            }
   {       dj        task j duration                            }

alias(y,yh,yf);
   {  y              fiscal year                                 }
   {    yh           historical fiscal year                      }
   {       yf        project finish fiscal year                  }

SETS
 budget_years(y,yf) { dynamic (static) set of budget fiscal years for each project finish year }
 idp3tuple(i,d,p)   { dynamic (static) set of tasks and admissable durations and work period   }
 isd3tuple(i,s,d)   { dynamic (static) set of tasks and admissable start times and durations   }
 jsd3tuple(i,s,d)   { dynamic (static) set of tasks and admissable start times and durations   }
;
FILE     logFile / fcs17_baseline.log /
PUT      logFile
SCALARS
 sampleSize
 counter
 cardp
 lb
 ub
 discount
 total
 dShape
 dSum
 dSum2
 dSample
 ml
 budgetShape
 maxFinishTime
 ScalingConstant
 costGrowthFactor
 start
 duration
 inflate
 uniDraw1
 uniDraw2
 delaytemp
 activitiesConsideredForDelay
 activitiesActuallyDelayed
;
SCALARS rowval,rowlow,rowup,cumslk ;
IF( CARD(m)<>12*CARD(y),
  PUT 'fiscal years in planning horizon do not reconcile with planning months...' / ;
  PUT '  planning months:',(CARD(m)) / ;
  PUT '  fiscal years:   ',(CARD(y)) / ;
  PUT '  fiscal months   ',(CARD(y)*12) / ;
);
```

```
 budgetShape = -log(0.03);
 ScalingConstant = 0.97 ;
 maxFinishTime = 0;
 costGrowthFactor = 0;
 counter = 0;
 sampleSize = 10000;
 dSum = 0;
 dSum2 = 0;
 dSample = 0;
 dShape = 1;
 total=0;
 delaytemp = 0;
 activitiesConsideredForDelay = 0;
 activitiesActuallyDelayed = 0;

 PARAMETERS
   es(i)
   ls(i)
   task_done(i)
   cost(i,d,p)  "cost of task i running for duration d months during month p"
   dMin(i)      "Minimum duration of activity i"
   dMax(i)      "Maximum duration of activity i"
   fp(d,p)
   dScale(i)    "Weibull Scale parameter used for activity determining durations"
   dMaxUB(i)    " Copy of dMax for use in moving window comparisons"
   dLocation(i) "Weibull Location parameter used for activity determining durations"
   dMode(i)
   dMean(i)
   dStdDev(i)
   activityStartTime(i,mw)  "Selected Activity start time at each iteration of moving window"
   activityDuration(i,mw)  "Selected Activity duration at each iteration of moving window"
   projectFinishTime(mw)    "Project finish time in a moving window iteration"
   simulationUB(mw)         "Solution upper Bound at each simulation iteration"
   simulationLB(mw)         "Solution lower Bound at each simulation iteration"
   results(mw,i,s,d)        "Array to collect the results"
;


 TABLE lookup_values(r,*)
$ondelim
$include CAIG_lookup_values.csv
$offdelim
;

$ontext
 ORIGINAL VERSION
 TABLE probability(r,*)
            delayProb    delayGrowth    costProb    costGrowth
   r1 {high}  0.6          1.8           0.8         1.8
   r2 {med }  0.4          1.4           0.6         1.4
   r3 {low }  0.2          1.1           0.4         1.1
 ;

LATEST VERSION
 TABLE probability(r,*)
            delayProb    delayGrowth    costProb    costGrowth
   r1 {high}  0.5          1.8           0.5         1.8
   r2 {med }  0.3          1.4           0.3         1.4
   r3 {low }  0.2          1.1           0.2         1.1
 ;

MID-WAY VERSION
$offtext
 TABLE probability(r,*)
            delayProb    delayGrowth    costProb    costGrowth
   r1 {high}  0.5          1.4           0.5         1.5
   r2 {med }  0.3          1.2           0.3         1.3
   r3 {low }  0.2          1.1           0.2         1.1
 ;
```

```
 ub=0 ;
 LOOP((y,fm),
   ub=ub+1 ;
 );

 IF( ub<>CARD(m),
   PUT 'internal set domain error ub= ',ub:5:0,'  CARD(p)= ',CARD(m):5:0 / ;
 );

 TABLE task_data(i,*)
$ondelim
$include FCS_tasks_simple.csv
$ontext
$include toy_task_data.csv
$offtext
$offdelim
 ;
*++++++++++++++++++++++++++++++++++++++++++++++++++
* Calculate and Assign values for dMin and dMax
ml = 0;
dShape = 0;
LOOP(i,
   loop(r$(ord(r) = task_data(i,"Risk") ),
      dShape = lookup_values( r ,"shape" );
      ml = lookup_values( r ,"ml");
   );
   dMode(i) = task_data(i,"Mode");
   if(dMode(i) = 0,
      dMin(i) = 0;
      dLocation(i) = 0;
      dMax(i) = 0;
      dMean(i) = 0;
      dStdDev(i) = 0;
   ELSE
      dLocation(i) = ceil(dMode(i)/ml);
      dScale(i) = ( dMode(i)- dMin(i) ) / (1-(1/dShape))**(1/dShape);

*     Now calculate the standard deviation and mean of each activities distribution.
*     First collect a sample
      FOR(counter = 1 TO sampleSize,
         dSample = ceil( dLocation(i) + dScale(i) *( (-log(UNIFORM(0,0.95)))**( 1 / dShape) ));
         dSum = dSum + dSample;
         dSum2 = dSum2 + dSample**2;
      );
*     Calculate the mean and StdDev
      dMean(i) = dSum / sampleSize;
      dStdDev(i) = SQRT( (sampleSize*dSum2 - (dSum**2))/sampleSize**2);

      dMin(i) = dLocation(i) + UNIFORM(0,0.1)*dStdDev(i);
      dMax(i) = dMode(i)     + UNIFORM(0,0.2)*dStdDev(i);

$ontext
      PUT 'non Zero duration, i = 'ORD(i):3:0 /;
      PUT 'dMode of   = 'dMode(i):8:4/;
      PUT 'dMean of   = 'dMean(i):8:4/;
      PUT 'dStdDev of = 'dStdDev(i):8:4 /
      PUT 'dLocation  = 'dLocation(i):8:4/;
      PUT 'dShape     = 'dShape:8:4/;
      PUT 'dScale     = 'dScale(i):8:4/;
      PUT 'dMin of    = 'dMin(i):8:4/;
      PUT 'dMax of    = 'dMax(i):8:4/
      PUT 'Old dMax   = 'ceil( dLocation(i) + dScale(i) *(-(log(0.95)))**( 1 / dShape) ):8:4/ /;
$offtext
      dSum = 0;
      dSum2 = 0;
      dShape = 0;
    );
*Close loop on i
);
```

```
*  Now convert from Weeks to Months.
{ wag: scale from weeks to months }
 LOOP(i,
   dMin(i) = FLOOR(dMin(i)/4);
   dMax(i) = CEIL(dMax(i)/4);
 );


*++++++++++++++++++++++++++++++++++++++++++++++++++
*  Create dynamic set of pairwise partial orders, including ultimate successor
 LOOP(i$(ORD(i)<>CARD(i)),
   LOOP(static_arcs(i,j),
     arcs(i,j)=yes ;
   );
   arcs(i,"last_task")=yes ;
 );

*++++++++++++++++++++++++++++++++++++++++++++++++++
*  LP Setup

 VARIABLES
   Z
  ;
 POSITIVE VARIABLES
   MONTH_MIN(i)         start month for MIN
   MONTH_MAX(i)         start month for MAX
   UNDER_CUM_BUDGET(y)
   SLACK_CUM_BUDGET(y)
   OVER_CUM_BUDGET (y)
 ;
 BINARY VARIABLES
   X(i,s,d)         start task i in month s with duration d months
   Q(yf)            finish project in fiscal year yf
 ;
 EQUATIONS
   MINI_MAX_TIME    earliest month project can complete
   ESIJ(i,j)        reverse star precedence using Min Time

   MAX_MINI_TIME    Maximum of soonest completions
   LSIJ(i,j)        forward star precedence using Min Time
 ;

*  FIND LOWER BOUND
   MINI_MAX_TIME..
     Z =e= SUM(j$(ORD(j)=CARD(j)),MONTH_MIN(j)+dMin(j))
   ;
   ESIJ(arcs(i,j))..
     MONTH_MIN(j) =g= MONTH_MIN(i) + dMin(i)
   ;
*  FIND UPPER BOUND
   MAX_MINI_TIME..
     Z =e= SUM(i$(ORD(i)=1),MONTH_MAX(i))
   ;
   LSIJ(arcs(i,j))..
     MONTH_MAX(i) =l= MONTH_MAX(j) - dMin(j)
    ;

 MODEL ESTART
 /
   MINI_MAX_TIME
   ESIJ
 / ;

 MODEL LSTART
 /
   MAX_MINI_TIME
   LSIJ
 / ;

 LOOP(i,
   MONTH_MIN.lo(i)=1 ;
```

```
 );

 SOLVE ESTART USING LP MINIMIZING Z ;
 lb=CEIL(Z.l);
 LOOP(j$(ORD(j)=CARD(j)),
   MONTH_MAX.up(j)=ub-dMIN(j) ;
 );
 LOOP(i,
   MONTH_MAX.lo(i)=1 ;
 );
 SOLVE LSTART USING LP MAXIMIZING Z ;
 PUT 'each task has slack of at least= ',(ub-lb):5:0 / ;
 PUT 'task slack in excess of this minimum...' / ;
 PUT 'task,dMin,slack,excess_slack' / ;
 LOOP(i,
   PUT i.tl,dMin(i):5:0,(MONTH_MAX.l(i)-MONTH_MIN.l(i)):5:0,(MONTH_MAX.l(i)-MONTH_MIN.l(i)-(ub-
        lb)):5:0 ;
   IF( MONTH_MAX.l(i)-MONTH_MIN.l(i)-(ub-lb)<=0,
     PUT ' <== on minimal task slack path' ;
   );
   PUT / ;
 );
*++++++++++++++++++++++++++++++++++++++++++++++++++++
* Update lower and upper bounds on project based on the LP relaxation.
 IF( ub<lb,
   PUT 'planning horizon of',ub:5:0,' weeks is shorter than critical path length',lb:5:0 / ;
 );
 LOOP(i,
   es(i) = MONTH_MIN.l(i) ;
 );
 LOOP(i,
   IF( MONTH_MAX.l(i) > MONTH_MIN.l(i) ,
     ls(i) = MONTH_MAX.l(i) ;
   ELSE
     ls(i) = MONTH_MIN.l(i);
   );
 );

put'lb,ub ',lb,ub / ;
 loop(i,
   dMax(i) = MAX(dMin(i),dMax(i)) ;
   dMaxUB(i) = dMax(i) * 2;
 );


 TABLE budget_data(y,yf,*)
$ondelim
$include fcs_budget_data.csv
$offdelim
 ;
 LOOP(yf$(SUM(y,ABS(budget_data(y,yf,"min"))+ABS(budget_data(y,yf,"max")))>0),
   IF(    ORD(yf)<CEIL(lb/12),
     PUT 'budget_data found for project finish year ',yf.tl:12,' prior to earliest year' / ;
     PUT '  (finish year budget ignored)' / ;
   ELSEIF ORD(yf)>CEIL(ub/12),
     PUT 'budget_data found for project finish year ',yf.tl:12,' following latest year' / ;
     PUT '  (finish year budget ignored)' / ;
   ELSE
     LOOP(y$(ABS(budget_data(y,yf,"min"))+ABS(budget_data(y,yf,"max"))>0),
       IF( ORD(y)>ORD(yf),
         PUT 'budget_data found for fiscal year beyond project finish year:' / ;
         PUT '  fiscal year:         ',y.tl:12 / ;
         PUT '  project finish year: ',yf.tl:12 / ;
         PUT '(entry ignored)' / ;
       ELSE
         budget_years(y,yf)=YES ;
       );
     );
   );
 );
 LOOP(yf$(SUM(budget_years(y,yf),1)>0),
```

```
   PUT 'BUDGET FOR PROJECT FINISH YEAR ',yf.tl:12 /;
   PUT '                 MIN    PLAN     MAX'/;
   LOOP(budget_years(y,yf),
     PUT y.tl:12 ;
     PUT budget_data(y,yf,"min"):8:0;
     PUT budget_data(y,yf,"plan"):8:0;
     PUT budget_data(y,yf,"max"):8:0 /;
   );
 );
*+++++++++++++++++++++++++++++++++++++++++++++++++++
*  Create the dynamic set of feasible tuples for use in the ILP.
 LOOP(yf$(SUM(budget_years(y,yf),1)>0),
   IF( ORD(yf)=CEIL(lb/12),
     Q.l(yf)=1 ;
     PUT 'heuristic starting solution: fastest project completion in ',yf.tl:6 / ;
   ELSE
     Q.l(yf)=0 ;
   );
 );
 PUT 'task                start    duration' / ;
 LOOP(i,
   LOOP(s$(ORD(s)>=es(i) and ORD(s)<=ls(i)),
     LOOP(d$(ORD(d)-1>=dMin(i) and ORD(d)-1<=MIN( dMax(i),CARD(m)-ORD(s)+1)),
       isd3tuple(i,s,d)=YES ;
       jsd3tuple(i,s,d)=YES ;
       IF( ORD(s)=es(i) and ORD(d)-1=dMin(i),
         X.l(i,s,d)=1 ;          { set candidate solution at earliest start, shortest duration }
         PUT i.tl:12,s.tl:12:0,(ORD(d)-1):12:0 / ;
       ELSE
         X.l(i,s,d)=0 ;
       );
     );
   );
 );
 LOOP(i,
   LOOP(d$(ORD(d)-1 >= dMin(i) and ORD(d)-1 <= 2*dMax(i)),   { heuristic, allowing at most
doubling of dMAX duration by annual review(s) }
     LOOP(p$(ORD(p)<=ORD(d)-1),
       idp3tuple(i,d,p)=yes ;
     );
   );
 );

* Output task information.
 PUT 'task i             es(i)       ls(i)    ex_slack     dMin(i)     dMax(i)     cost(i)
factor(i)' / ;
 LOOP(i,
   PUT i.tl:12,es(i):12:1,ls(i):12:1 ;
   PUT (MONTH_MAX.l(i)-MONTH_MIN.l(i)-(ub-lb)):12:1 ;
   PUT dMin(i):12:0,dMax(i):12:0 ;
   PUT task_data(i,"cost"):12:3,task_data(i,"factor"):12:2 / ;
 );
 PUT 'task i        duration d    period p cost(i,d,p)  discounted' / ;

$ontext
  the dynamic set idp3tuple of feasible 3-tuples has been created with
  exactly one pass of the indexes i, d, and p, with no subsequent filtering
  accordingly, the 3-tuples in this set should be in hierarchical order
  with p varying fastest
$offtext

*+++++++++++++++++++++++++++++++++++++++++++++++++++
*  Spread Cost of activity i for each of the feasible 3-tuples.
 LOOP(d$(ORD(d)<=2*SMAX(i,dMax(i))),
   LOOP(p$(ORD(p)<=ORD(d)-1),
     fp(d,p)=exp(-(budgetShape/power(ord(d)-1,2))*power(ord(p)-1,2))
            -exp(-(budgetShape/power(ord(d)-1,2))*power(ord(p)  ,2));
   );
 );
 LOOP(idp3tuple(i,d,p),
   discount=exp((ORD(d)-1-dMin(i))* task_data(i,"factor"));
```

```
*  Now calculate cost and allocate to appropriate 3-tuple
   cost(i,d,p)=discount*(task_data(i,'cost')/ScalingConstant)*(fp(d,p)) ;
   IF( ORD(p)=1,
     PUT i.tl:12,(ORD(d)-1):12:0,'            ' ;
     PUT task_data(i,"cost"):12:3,(discount*task_data(i,"cost")):12:3 / ;
   );
   PUT '            ','            ',ORD(p):12:0,cost(i,d,p):12:3 ;
   total=total+cost(i,d,p) ;
   IF( ORD(p)=ORD(d)-1,
     PUT total:12:3 ;
     total = 0;
   );
   PUT / ;
 );

*+++++++++++++++++++++++++++++++++++++++++++++++++
* ILP setup
 EQUATIONS
   PROJECT_MONTHS          objective function
   USE_1(i)                partition constraint
   FINISH_IN_yf(yf,i,s,d)  variable upper bound constraint
   JUST_1_FINISH           partition constraint
   CUM_FY_BUDGET(y)        fiscal year budget constraint
   SLACK_up(y)             upper bound on cumulative slack budget in fiscal year y
   ORDER(i,j,j,sj,dj)      any task j start must follow some predecessor task i finish
 ;
*  (F1) objective function)
   PROJECT_MONTHS..
     Z =e= SUM(isd3tuple(i,s,d)$(ORD(i)=CARD(i)),(ORD(s)+ORD(d)-1)*X(i,s,d))
         + SUM(y,0.1*UNDER_CUM_BUDGET(y) + 1.0*OVER_CUM_BUDGET(y))
   ;

*  (F2)
   USE_1(i)..
     SUM(isd3TUPLE(i,s,d),X(i,s,d)) =e= 1
   ;

*  (F3)
   FINISH_IN_yf(yf,isd3tuple(i,s,d))$(SUM(budget_years(y,yf),1)>0
                                  and ORD(i)=CARD(i)
                                  and SUM(m$(ORD(m)>=CARD(fm)*(ORD(yf)-1)+1
                                         and ORD(m)<=CARD(fm)*ORD(yf)
                                         and ORD(s)+ORD(d)-1=ORD(m)),1)>0)..
     X(i,s,d) =l= Q(yf)
   ;

*  (F4)
   JUST_1_FINISH..
     SUM(yf$(SUM(budget_years(y,yf),1)>0),Q(yf)) =e= 1
   ;

*  (F5)
   CUM_FY_BUDGET(y)..
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                        and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X(i,s,d)
       )
     )
     +UNDER_CUM_BUDGET(y)+SLACK_CUM_BUDGET(y)-OVER_CUM_BUDGET(y)
       =e= SUM(budget_years(y,yf)$(ORD(y)<=ORD(yf)),
             budget_data(y,yf,"max")*Q(yf)
           )
           +(UNDER_CUM_BUDGET(y-1)+SLACK_CUM_BUDGET(y-1)-OVER_CUM_BUDGET(y-1))$(ORD(y)>=2)
   ;

*  (F6) upper bound on cumulative budget slack through fiscal year y
   SLACK_up(y)..
     SLACK_CUM_BUDGET(y) =l= SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q(yf))
```

```
   ;

*  (F7) any task j start must follow some predecessor task i finish
   ORDER(arcs(i,j),jsd3tuple(j,sj,dj))$(ORD(sj)>=MAX(es(j),es(i)+dMin(i)))..
     SUM(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),X(i,si,di))
       =g= X(j,sj,dj)
   ;



* begin do-it-yourself preprocessor

*test initial incumbent

$ontext
*  (F2)
   USE_1(i)..
     SUM(isd3TUPLE(i,s,d),X(i,s,d)) =e= 1
   ;
$offtext
   LOOP(i,
     IF(
     SUM(isd3TUPLE(i,s,d),X.l(i,s,d)) <> 1,
       PUT 'USE_1 ',i.tl / ;
       LOOP(isd3TUPLE(i,s,d)$(X.l(i,s,d)>0),
         put ' i,s,d,x.l(i.s.d) ',i.tl,s.tl,d.tl,X.l(i,s,d) / ;
       );
     );
   );
$ontext
*  (F3)
   FINISH_IN_yf(yf,isd3tuple(i,s,d))$(SUM(budget_years(y,yf),1)>0
                                and ORD(i)=CARD(i)
                                and SUM(m$(ORD(m)>=CARD(fm)*(ORD(yf)-1)+1
                                       and ORD(m)<=CARD(fm)*ORD(yf)
                                       and ORD(s)+ORD(d)-1=ORD(m)),1)>0)..
     X(i,s,d) =l= Q(yf)
   ;
$offtext
   LOOP(      (yf,isd3tuple(i,s,d))$(SUM(budget_years(y,yf),1)>0
                                and ORD(i)=CARD(i)
                                and SUM(m$(ORD(m)>=CARD(fm)*(ORD(yf)-1)+1
                                       and ORD(m)<=CARD(fm)*ORD(yf)
                                       and ORD(s)+ORD(d)-1=ORD(m)),1)>0),
     IF( X.l(i,s,d)>Q.l(yf),
       PUT 'FINISH_IN_yf ',yf.tl,i.tl,s.tl,d.tl / ;
       PUT 'X.l(i,s,d),Q.l(yf) ',X.l(i,s,d),Q.l(yf) / ;
     );
   );
$ontext
*  (F4)
   JUST_1_FINISH..
     SUM(yf$(SUM(budget_years(y,yf),1)>0),Q(yf)) =e= 1
   ;
$offtext
   IF(
     SUM(yf$(SUM(budget_years(y,yf),1)>0),Q.l(yf)) <> 1,
       PUT 'JUST_1_FINISH' / ;
       LOOP(yf$(SUM(budget_years(y,yf),1)>0),
         PUT 'yf.l,Q.l(yf) ',yf.tl,Q.l(yf) / ;
       );
   );
$ontext
*  (F5)
   CUM_FY_BUDGET(y)..
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                         and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X(i,s,d)
       )
```

```
      )
      +UNDER_CUM_BUDGET(y)+SLACK_CUM_BUDGET(y)-OVER_CUM_BUDGET(y)
        =e= SUM(budget_years(y,yf)$(ORD(y)<=ORD(yf)),budget_data(y,yf,"max")*Q(yf))
          +(UNDER_CUM_BUDGET(y-1)+SLACK_CUM_BUDGET(y-1)-OVER_CUM_BUDGET(y-1))$(ORD(y)>=2)
   ;
*  (F6) upper bound on cumulative budget slack through fiscal year y
   SLACK_up(y)..
     SLACK_CUM_BUDGET(y) =l= SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q(yf))
   ;
$offtext
   LOOP(         y,
     PUT 'CUM_FY_BUDGET ',y.tl / ;
     UNDER_CUM_BUDGET.l(y)=0 ;
     SLACK_CUM_BUDGET.l(y)=0 ;
     OVER_CUM_BUDGET.l(y)=0 ;
     cumslk=
     SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))
     ;
     rowval=
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                         and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
       )
     )
          -SUM(budget_years(y,yf)$(ORD(y)<=ORD(yf)),
             budget_data(y,yf,"max")*Q.l(yf)
           )
           -(+UNDER_CUM_BUDGET.l(y-1)+SLACK_CUM_BUDGET.l(y-1)-OVER_CUM_BUDGET.l(y-1))$(ORD(y)>=2)
     ;


     PUT y.tl,rowval / ;
     IF( rowval>=0,
       OVER_CUM_BUDGET.l(y)=rowval ;
       PUT 'over  ',OVER_CUM_BUDGET.l(y) / ;
     ELSEIF -rowval<=cumslk,
       SLACK_CUM_BUDGET.l(y)=-rowval ;
       PUT 'slack ',SLACK_CUM_BUDGET.l(y) / ;
     ELSE
       UNDER_CUM_BUDGET.l(y)=-rowval-cumslk ;
       SLACK_CUM_BUDGET.l(y)=cumslk ;
       PUT 'under ',UNDER_CUM_BUDGET.l(y) / ;
       PUT 'slack ',SLACK_CUM_BUDGET.l(y) / ;
     );



IF( ORD(y)>=2,
  PUT 'bal forward ',
          (-(UNDER_CUM_BUDGET.l(y-1)-SLACK_CUM_BUDGET.l(y-1)+OVER_CUM_BUDGET.l(y-1)))
  /;
);
PUT 'spent ',
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                         and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
       )
     )
/;
PUT 'bal forward ',
        (-(UNDER_CUM_BUDGET.l(y)-SLACK_CUM_BUDGET.l(y)+OVER_CUM_BUDGET.l(y)))
/;
```

```
   );


   LOOP(    y,
     IF( SLACK_CUM_BUDGET.l(y) > SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf)),
       PUT 'SLACK_up ',y.tl / ;
       PUT 'SLACK ',SLACK_CUM_BUDGET.l(y) / ;
       PUT 'CUM   ',(SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))) / ;
     );
   );
   LOOP(y,
     PUT y.tl,'under,slack,over' / ;
     IF( ORD(y)>=2,
       PUT UNDER_CUM_BUDGET.l(y-1);
       PUT SLACK_CUM_BUDGET.l(y-1);
       PUT OVER_CUM_BUDGET.l(y-1);
     ELSE
       PUT 0 ;
       PUT 0 ;
       PUT 0 ;
     );
     PUT / ;
     PUT 'spent ',
     (SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                         and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
       )
     ))/;
     PUT 'max slack';
     PUT(SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))
     )/;
     PUT UNDER_CUM_BUDGET.l(y);
     PUT SLACK_CUM_BUDGET.l(y);
     PUT OVER_CUM_BUDGET.l(y)/;
   );

$ontext
*  (F7) any task j start must follow some predecessor task i finish
   ORDER(arcs(i,j),jsd3tuple(j,sj,dj))$(ORD(sj)>=MAX(es(j),es(i)+dMin(i)))..
     SUM(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),X(i,si,di))
       =g= X(j,sj,dj)
   ;
$offtext
   LOOP((arcs(i,j),jsd3tuple(j,sj,dj))$(ORD(sj)>=MAX(es(j),es(i)+dMin(i))),
     IF(
     SUM(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),X.l(i,si,di))

        <  X.l(j,sj,dj),
       PUT 'ORDER i,j,sj,dj',i.tl,j.tl,sj.tl,dj.tl / ;
       LOOP(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),
         PUT 'i,si,di,X ',i.tl,si.tl,di.tl,X.l(i,si,di) / ;
       );
       PUT 'j,sj,dj,x ',j.tl,sj.tl,dj.tl,X.l(j,sj,dj) / ;
     );
   );

* end do-it-yourself preprocessor



 MODEL FCS_SCHEDULER
 /
   PROJECT_MONTHS
   USE_1
   FINISH_IN_yf
```

```
   JUST_1_FINISH
   CUM_FY_BUDGET
   SLACK_up
   ORDER
 / ;




 FCS_SCHEDULER.optfile = 1;
*  Solve once. This gives the case with no sliding window.
$ontext
 FCS_SCHEDULER.prioropt = 1;
 Q.prior(yf)=1;
 X.prior(i,s,d)=2;
$offtext

* I have increased the reslim as will be running the model overnight.
 FCS_SCHEDULER.reslim=800 ;

 SOLVE FCS_SCHEDULER USING MIP MINIMIZING Z ;

 IF (FCS_SCHEDULER.modelstat <> 1 AND FCS_SCHEDULER.modelstat <> 8,
    PUT '++++ Error solving model.  model status = 'FCS_SCHEDULER.modelstat:3:0/;
 ELSE
    PUT /' Best Upper Bound = 'Z.l:10:4 / ;
    PUT /' Best Lower Bound = 'FCS_SCHEDULER.objest:10:4 / ;
 );


 LOOP(i,
   LOOP(isd3tuple(i,s,d)$(ORD(i)=CARD(i)),
     PUT 'i.tl,s.tl,d.tl,x(i,s,d) ',i.tl,s.tl,d.tl,' ',x.l(i,s,d) / ;
     PUT '((ORD(s)+ORD(d)-1-1)*X.l(i,s,d)) ',((ORD(s)+ORD(d)-1-1)*X.l(i,s,d)) / ;
   );
 );
 LOOP(y,
   IF( UNDER_CUM_BUDGET.l(y)>0,
     PUT '0.1*UNDER_CUM_BUDGET(y) ',y.tl:12,(0.1*UNDER_CUM_BUDGET.l(y)) / ;
   );
   IF( OVER_CUM_BUDGET.l(y)>0,
     PUT '1.0*OVER_CUM_BUDGET(y)  ',y.tl:12,(1.0*OVER_CUM_BUDGET.l(y)) / ;
   );
 );

 PUT 'finish last task in month ',(SUM(isd3tuple(i,s,d)$(ORD(i)=CARD(i)),(ORD(s)+ORD(d)-
      1)*X.l(i,s,d))) / ;
 LOOP(yf$(Q.l(yf)=1),
   PUT 'finish project in fiscal year ',yf.tl:12 / ;
 );
*+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
*  Start the moving month window which move through the periods
*  and randomizes activities not yet started.  It then resolves and
*  this solution is used as the next baseline.
 PUT 'Starting Moving Window Loop' / ;

 LOOP(i,
   task_done(i)=0 ;
 );

* Loop over all time periods.  Alias of Month called mw (month window)
 LOOP(mw$(MOD(ORD(mw),12)=0 and ORD(mw)<0),
   PUT 'annual review, month ',mw.tl:12 / ;
   LOOP(i,
     LOOP(s$(ORD(s)>=es(i) and ORD(s)<=ls(i)),
       LOOP(d$(ORD(d)-1>=dMin(i) and ORD(d)-1<=MIN(dMax(i),ub-ORD(s)+1)),
         isd3tuple(i,s,d)=NO ;
         jsd3tuple(i,s,d)=NO ;
       );
     );
   );
```

```
put 'check for empty dynamic sets' / ;
LOOP(isd3tuple(i,s,d), put 'isd leaker: ',i.tl,s.tl,d.tl / ; );

   PUT 'Cleared out Dynamic sets.  Looking for the new feasible sets...'/;

*  Loop over all tasks not already marked completed
   LOOP(i$( task_done(i)=0 ),
*     Loop over all feasible start times
      inflate=1 ;
      LOOP(s$(ORD(s)>=es(i) and ORD(s)<=ls(i) ),
*       not yet started
*       Loop over all feasible durations
        LOOP(d$( ORD(d)-1 >= dMin(i) and ORD(d)-1 <= MIN(dMax(i),ub-ORD(s)+1) and X.l(i,s,d) = 1
),
          IF( ORD(s)+ORD(d)-1 <= ORD(mw),
*           task completed since last annual review
            task_done(i)=ORD(s)+ORD(d)-1 ;
            start=ORD(s) ;
            duration=ORD(d)-1 ;
            put 'task completed since last annual review ',i.tl,s.tl,d.tl,task_done(i):5:0 / ;
          ELSEIF ORD(s) <= ORD(mw),
*           task still in progress at this annual review
            put 'task annual review ',i.tl,s.tl,d.tl,task_data(i,"risk"):5:0 / ;
            start=ORD(s) ;
            duration=(ORD(d)-1) ;


            LOOP(r$(ord(r)=task_data(i,"Risk")),
              uniDraw1=UNIFORM(0,1) ;
              activitiesConsideredForDelay = activitiesConsideredForDelay + 1;
              PUT 'uniDraw1 = 'uniDraw1:5:4 ' at i,s,d = ',ORD(i):5:1,ORD(s):5:1,ORD(d):5:0 /;
              IF( uniDraw1<=probability(r,'delayProb'),
                activitiesActuallyDelayed = activitiesActuallyDelayed + 1;
*               Check to see if the delay is less than twice the original dMax(i)
                delayTemp = CEIL(duration*probability(r,'delayGrowth'))
                IF(delayTemp <= dMaxUB(i),
                  duration=delayTemp;
                  put '  *** delaying task ',i.tl,' by delayTemp = ',delayTemp /;
                  put '      dMaxUB(i) = ',dMaxUB(i)/;
                  put '      task ',i.tl,'  duration delayed from ',d.tl,'  to ',duration:5:0 / ;
                ELSE
                  duration = dMaxUB(i);
                  put '  task ',i.tl,'  maximally delayed at  ',d.tl/ ;
                );
                IF( ORD(s)+duration <= ORD(mw),
*                 despite delay, task completed since last annual review
                  task_done(i)=ORD(s)+duration ;
                  put '  delayed task completed since last annual review
                    ',i.tl,s.tl,d.tl,duration:5:0,task_done(i):5:0 / ;
                );
                uniDraw2=UNIFORM(0,1) ;
                IF( uniDraw2<=probability(r,'costProb'),
                  inflate=probability(r,'costGrowth') ;
                );
              );
            );

          ELSE
*           task not yet started by this annual review
            duration=-1 ;
          );
        );
      );
      IF( task_done(i)>0,
*       task marked completed during this annual review
        es(i)=start ;
        ls(i)=start ;
        put 'completed ',es(i),ls(i) / ;
      );
      IF( duration>=0,
```

```
        dMin(i)=duration ;
        dMax(i)=duration ;
        LOOP(s$( ORD(s)=start ),
          LOOP(d$(ORD(d)-1=duration),
            X.lo(i,s,d)=1 ;
          );
        );
      );
      IF( inflate>1,
*       apply inflation to remaining (future) costs
        LOOP(idp3TUPLE(i,d,p)$(ORD(d)-1=duration and ORD(p)>ORD(mw)+1-start),
          cost(i,d,p)=inflate*cost(i,d,p) ;
          put 'i.tl,start,d,tl,p.tl,inflate,cost ',i.tl,start:5:0,'
',d.tl,p.tl,inflate:5:1,cost(i,d,p) / ;
        );
      );
    );
    LOOP((y,yf),
      budget_years(y,yf)=NO ;
    );
    LOOP(yf$(SUM(y,ABS(budget_data(y,yf,"min"))+ABS(budget_data(y,yf,"max")))>0),
      IF(    ORD(yf)<CEIL(lb/12),
        PUT 'budget_data found for project finish year ',yf.tl:12,' prior to earliest year' / ;
        PUT '  (finish year budget ignored)' / ;
      ELSEIF ORD(yf)>CEIL(ub/12),
        PUT 'budget_data found for project finish year ',yf.tl:12,' following latest year' / ;
        PUT '  (finish year budget ignored)' / ;
      ELSE
        LOOP(y$(ABS(budget_data(y,yf,"min"))+ABS(budget_data(y,yf,"max"))>0),
          IF( ORD(y)>ORD(yf),
            PUT 'budget_data found for fiscal year beyond project finish year:' / ;
            PUT '  fiscal year:         ',y.tl:12 / ;
            PUT '  project finish year: ',yf.tl:12 / ;
            PUT '(entry ignored)' / ;
          ELSE
            budget_years(y,yf)=YES ;
          );
        );
      );
    );
    LOOP(yf$(SUM(budget_years(y,yf),1)>0),
      PUT 'BUDGET FOR PROJECT FINISH YEAR ',yf.tl:12 /;
      PUT '                  MIN    PLAN     MAX'/;
      LOOP(budget_years(y,yf),
        PUT y.tl:12 ;
        PUT budget_data(y,yf,"min"):8:0;
        PUT budget_data(y,yf,"plan"):8:0;
        PUT budget_data(y,yf,"max"):8:0 /;
      );
    );

    FCS_SCHEDULER.optfile = 1;
    FCS_SCHEDULER.reslim=500 ;
    SOLVE ESTART USING LP MINIMIZING Z ;
    lb=CEIL(Z.l);
*   Create the dynamic set of feasible tuples for use in the ILP.
    LOOP(yf$(SUM(budget_years(y,yf),1)>0),
      IF( ORD(yf)=CEIL(lb/12),
        Q.l(yf)=1 ;
        PUT 'heuristic starting solution: fastest project completion in ',yf.tl:6 / ;
      ELSE
        Q.l(yf)=0 ;
      );
    );
    PUT 'task                start    duration' / ;
    LOOP(i,
      LOOP(s$(ORD(s)>=es(i) and ORD(s)<=ls(i)),
        LOOP(d$(ORD(d)-1>=dMin(i) and ORD(d)-1<=MIN(dMax(i),CARD(m)-ORD(s)+1)),
          isd3tuple(i,s,d)=YES ;
          jsd3tuple(i,s,d)=YES ;
          IF( ORD(s)=es(i) and ORD(d)-1=dMin(i),
```

```
          X.l(i,s,d)=1 ;         {set candidate solution at earliest start, shortest duration}
          PUT i.tl:12,s.tl:12:0,(ORD(d)-1):12:0 / ;
        ELSE
          X.l(i,s,d)=0 ;
        );
      );
    );
  );

  PUT ' Finished finding Feasible sets'/;
  PUT ' Ready to Solve the ILP'/;
  PUT 'check isd3TUPLE(i,s,d) task-by-task ' / ;
  LOOP(i,
    PUT i.tl,' alternatives available= ',(SUM(isd3TUPLE(i,s,d),1)):4:0 / ;
  );


*  begin preprocessor
*  test initial incumbent
$ontext
*  (F2)
   USE_1(i)..
     SUM(isd3TUPLE(i,s,d),X(i,s,d)) =e= 1
   ;
$offtext
   LOOP(i,
     IF(
     SUM(isd3TUPLE(i,s,d),X.l(i,s,d)) <> 1,
       PUT 'USE_1 ',i.tl / ;
       LOOP(isd3TUPLE(i,s,d)$(X.l(i,s,d)>0),
         put ' i,s,d,x.l(i.s.d) ',i.tl,s.tl,d.tl,X.l(i,s,d) / ;
       );
     );
   );

$ontext
*  (F3)
   FINISH_IN_yf(yf,isd3tuple(i,s,d))$(SUM(budget_years(y,yf),1)>0
                                  and ORD(i)=CARD(i)
                                  and SUM(m$(ORD(m)>=CARD(fm)*(ORD(yf)-1)+1
                                         and ORD(m)<=CARD(fm)*ORD(yf)
                                         and ORD(s)+ORD(d)-1=ORD(m)),1)>0)..
     X(i,s,d) =l= Q(yf)
   ;
$offtext

   LOOP( (yf,isd3tuple(i,s,d))$(SUM(budget_years(y,yf),1)>0
                               and ORD(i)=CARD(i)
                               and SUM(m$(ORD(m)>=CARD(fm)*(ORD(yf)-1)+1
                                      and ORD(m)<=CARD(fm)*ORD(yf)
                                      and ORD(s)+ORD(d)-1=ORD(m)),1)>0),
     IF( X.l(i,s,d)>Q.l(yf),
       PUT 'FINISH_IN_yf ',yf.tl,i.tl,s.tl,d.tl / ;
       PUT 'X.l(i,s,d),Q.l(yf) ',X.l(i,s,d),Q.l(yf) / ;
     );
   );

$ontext
*  (F4)
   JUST_1_FINISH..
     SUM(yf$(SUM(budget_years(y,yf),1)>0),Q(yf)) =e= 1
   ;
$offtext

   IF(
     SUM(yf$(SUM(budget_years(y,yf),1)>0),Q.l(yf)) <> 1,
       PUT 'JUST_1_FINISH' / ;
       LOOP(yf$(SUM(budget_years(y,yf),1)>0),
         PUT 'yf.l,Q.l(yf) ',yf.tl,Q.l(yf) / ;
       );
   );
```

```
$ontext
*  (F5)
   CUM_FY_BUDGET(y)..
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                           and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X(i,s,d)
       )
     )
     +UNDER_CUM_BUDGET(y)+SLACK_CUM_BUDGET(y)-OVER_CUM_BUDGET(y)
       =e= SUM(budget_years(y,yf)$(ORD(y)<=ORD(yf)),budget_data(y,yf,"max")*Q(yf))
         +(UNDER_CUM_BUDGET(y-1)+SLACK_CUM_BUDGET(y-1)-OVER_CUM_BUDGET(y-1))$(ORD(y)>=2)
   ;
*  (F6) upper bound on cumulative budget slack through fiscal year y
   SLACK_up(y)..
     SLACK_CUM_BUDGET(y) =l= SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q(yf))
   ;
$offtext

   LOOP(         y,
     PUT 'CUM_FY_BUDGET ',y.tl / ;
     UNDER_CUM_BUDGET.l(y)=0 ;
     SLACK_CUM_BUDGET.l(y)=0 ;
     OVER_CUM_BUDGET.l(y)=0 ;
     cumslk=
     SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
       (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))
     ;
     rowval=
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                           and ORD(m)-ORD(s)+1<=ORD(d)-1),
         cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
       )
     )
          -SUM(budget_years(y,yf)$(ORD(y)<=ORD(yf)),
             budget_data(y,yf,"max")*Q.l(yf)
           )
           -(+UNDER_CUM_BUDGET.l(y-1)+SLACK_CUM_BUDGET.l(y-1)-OVER_CUM_BUDGET.l(y-1))$(ORD(y)>=2)
     ;


     PUT y.tl,rowval / ;
     IF( rowval>=0,
       OVER_CUM_BUDGET.l(y)=rowval ;
       PUT 'over  ',OVER_CUM_BUDGET.l(y) / ;
     ELSEIF -rowval<=cumslk,
       SLACK_CUM_BUDGET.l(y)=-rowval ;
       PUT 'slack ',SLACK_CUM_BUDGET.l(y) / ;
     ELSE
       UNDER_CUM_BUDGET.l(y)=-rowval-cumslk ;
       SLACK_CUM_BUDGET.l(y)=cumslk ;
       PUT 'under ',UNDER_CUM_BUDGET.l(y) / ;
       PUT 'slack ',SLACK_CUM_BUDGET.l(y) / ;
     );


   IF( ORD(y)>=2,
      PUT 'bal forward ',
             (-(UNDER_CUM_BUDGET.l(y-1)-SLACK_CUM_BUDGET.l(y-1)+OVER_CUM_BUDGET.l(y-1)))
     /;
   );
   PUT 'spent ',
     SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
       SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                           and ORD(m)-ORD(s)+1<=ORD(d)-1),
```

```
        cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
      )
    )
  /;
  PUT 'bal forward ',
       (-(UNDER_CUM_BUDGET.l(y)-SLACK_CUM_BUDGET.l(y)+OVER_CUM_BUDGET.l(y)))
  /;
  );

  LOOP(    y,
    IF( SLACK_CUM_BUDGET.l(y) > SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
      (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf)),
      PUT 'SLACK_up ',y.tl / ;
      PUT 'SLACK ',SLACK_CUM_BUDGET.l(y) / ;
      PUT 'CUM   ',(SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
      (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))) / ;
    );
  );
  LOOP(y,
    PUT y.tl,'under,slack,over' / ;
    IF( ORD(y)>=2,
      PUT UNDER_CUM_BUDGET.l(y-1);
      PUT SLACK_CUM_BUDGET.l(y-1);
      PUT OVER_CUM_BUDGET.l(y-1);
    ELSE
      PUT 0 ;
      PUT 0 ;
      PUT 0 ;
    );
    PUT / ;
    PUT 'spent ',
    (SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
       and ORD(m)<=CARD(fm)*ORD(y)),
      SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                      and ORD(m)-ORD(s)+1<=ORD(d)-1),
        cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
      )
    ))/;
    PUT 'max slack';
    PUT(SUM((yh,budget_years(yh,yf))$(ORD(yh)<=ORD(y)),
      (budget_data(yh,yf,"max") - budget_data(yh,yf,"min"))*Q.l(yf))
    )/;
    PUT UNDER_CUM_BUDGET.l(y);
    PUT SLACK_CUM_BUDGET.l(y);
    PUT OVER_CUM_BUDGET.l(y)/;
  );
  put 'ping a'

$ontext
*  (F7) any task j start must follow some predecessor task i finish
  ORDER(arcs(i,j),jsd3tuple(j,sj,dj))$(ORD(sj)>=MAX(es(j),es(i)+dMin(i)))..
    SUM(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),X(i,si,di))
      =g= X(j,sj,dj)
  ;
$offtext

  LOOP((arcs(i,j),jsd3tuple(j,sj,dj))$(ORD(sj)>=MAX(es(j),es(i)+dMin(i))),
    IF(
    SUM(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),X.l(i,si,di))

       <  X.l(j,sj,dj),
      PUT 'ORDER i,j,sj,dj',i.tl,j.tl,sj.tl,dj.tl / ;
      LOOP(isd3tuple(i,si,di)$(ORD(si)+ORD(di)-1<=ORD(sj)),
        PUT 'i,si,di,X ',i.tl,si.tl,di.tl,X.l(i,si,di) / ;
      );
      PUT 'j,sj,dj,X ',j.tl,sj.tl,dj.tl,X.l(j,sj,dj) / ;
    );
  );
  put 'ping a'

*  end do-it-yourself preprocessor
```

```
  SOLVE FCS_SCHEDULER USING MIP MINIMIZING Z ;

 IF (FCS_SCHEDULER.modelstat <> 1 AND FCS_SCHEDULER.modelstat <> 8,
    PUT '++++ Error solving model.  model status = 'FCS_SCHEDULER.modelstat:3:0/;
  ELSE
    simulationUB(mw) = Z.l;
    simulationLB(mw) = FCS_SCHEDULER.objest;
    PUT /' Best Upper Bound = 'simulationLB(mw):10:4 / ;
    PUT /' Best Lower Bound = 'simulationUB(mw):10:4 / ;
  );



  PUT 'finish last task in month ',(SUM(isd3tuple(i,s,d)$(ORD(i)=CARD(i)),(ORD(s)+ORD(d)-
       1)*X.l(i,s,d))) / ;
  LOOP(yf$(Q.l(yf)=1),
    PUT 'finish project in fiscal year ',yf.tl:12 / ;
  );

  PUT ' Finished solve.  Solution set is:' / ;
*  Display the task list with start and duration options.
  PUT ' task i             start    duration' / ;
  LOOP(isd3tuple(i,s,d)$(X.l(i,s,d)>0) ,
    activityStartTime(i,mw)=ORD(s) ;
    activityDuration(i,mw) =ORD(d) ;
    PUT i.tl:12,ORD(s):12:0,(ORD(d)-1):12:0 / ;
    IF((ORD(s)+ORD(d))>maxFinishTime ,
      maxFinishTime=ORD(s)+ORD(d) ;
    );
  );
  projectFinishTime(mw)=maxFinishTime ;
  maxFinishTime=0 ;

  PUT  / /'At end of Moving Window Iteration ' ORD(mw) / / ;
* end loop on mw
 );

 PUT / /'Finished the Time Step Simulation' / ;

 PUT '+++++++++++++++++++++++++++++++++++++++++++++++'/;

 LOOP(mw$(projectFinishTime(mw)>0),
   PUT 'projectFinishTime at iteration 'ORD(MW)' is 'projectFinishTime(mw) / ;
   PUT '   Best Lower Bound : ' simulationLB(mw)/;
   PUT '   Best Upper Bound : ' simulationUB(mw)/;
 );

 PUT '*** Activities Considered For Delay = 'activitiesConsideredForDelay:3:0/;
 PUT '*** Activities Actually Delayed     = 'activitiesActuallyDelayed:3:0/ /;


 PUT 'Output results for the final solve'/ /;
 LOOP(yf$(Q.l(yf)=1),
   PUT // 'cumulative budget for project finish in fiscal year ',yf.tl:12 / ;
   PUT '                year_min  year_spent    year_max     cum_min   cum_spent     cum_max
                       cum_slack' / ;
   LOOP(budget_years(y,yf),
     PUT y.tl:12,
     PUT budget_data(y,yf,"min"):12:3 ;
     PUT
    (SUM(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
      SUM(isd3tuple(i,s,d)$(ORD(m)-ORD(s)+1>=1
                        and ORD(m)-ORD(s)+1<=ORD(d)-1),
        cost(i,d,m-(ORD(s)-1))*X.l(i,s,d)
      )
     )):12:3;
     PUT budget_data(y,yf,"max"):12:3 ;
```

```
    PUT SUM(yh$(ORD(yh)<=ORD(y)),budget_data(yh,yf,"min")):12:3 ;
    PUT (SUM(yh$(ORD(yh)<=ORD(y)),budget_data(yh,yf,"max"))
      -UNDER_CUM_BUDGET.l(y)-SLACK_CUM_BUDGET.l(y)+OVER_CUM_BUDGET.l(y)):12:3 ;
    PUT (SUM(yh$(ORD(yh)<=ORD(y)),budget_data(yh,yf,"max"))):12:3 ;
    PUT SLACK_CUM_BUDGET.l(y) / ;
    IF( UNDER_CUM_BUDGET.l(y)>0,
      PUT '                                                             under_cum_min:
           ',UNDER_CUM_BUDGET.l(y):12:3 / ;
    );
    IF( SLACK_CUM_BUDGET.l(y)>0,
      PUT '                                                                 cum_slack:
           ',SLACK_CUM_BUDGET.l(y):12:3 / ;
    );
    IF( OVER_CUM_BUDGET.l(y)>0,
      PUT '                                                              over_cum_max:
          ',OVER_CUM_BUDGET.l(y):12:3 / ;
    );
  );
  PUT 'task i             start    duration' / ;
  LOOP(isd3tuple(i,s,d)$(X.l(i,s,d)>0),
    PUT i.tl:12,'  s ',s.tl:8,'  d ',(ORD(d)-1):8:0 / ;
  );
  PUT 'fiscal schedule for project finish in fiscal year ',yf.tl:12 / ;
  LOOP(y,
    PUT y.tl:12 / ;
    LOOP(m$(ORD(m)>=CARD(fm)*(ORD(y)-1)+1
        and ORD(m)<=CARD(fm)*ORD(y)),
      PUT '            ',m.tl:12 / ;
      LOOP(isd3tuple(i,s,d)$(X.l(i,s,d)>0),
        IF( ORD(m)-ORD(s)+1>=1 and ORD(m)-ORD(s)+1<=ORD(d)-1,
          PUT '                        ' ;
          PUT i.tl:12,'  s ',s.tl:8,'  d ',(ORD(d)-1):8:0,'  p ',(ORD(m)-ORD(s)+1):8:0 ;
          PUT cost(i,d,m-(ORD(s)-1)):12:3 / ;
        );
      );
    );
  );
);
PUTCLOSE logFile
```

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